Problem Set I, Part II
Advanced Macroeconomics II

Antai College of Economic and Management,
Shanghai Jiao Tong University
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Note: please submit before the beginning of the lecture on December 18, 2017. Late submission is NOT accepted.

Unit measure of workers and firms in the model economy. Each firm can hire at most one worker. We assume that the total size (or more exactly the mass) of the population is normalized to 1, which implies that the unemployment rate u is equal to the unemployment level U.

There are transaction costs in this economy, which imply that vacant jobs and unemployment workers coexist in equilibrium. The matching function determines the number of matches M realized by unit of time when there are V vacancies and U unemployed workers. We denote by $M(V, U)$ this matching function. We assume that this matching function is increasing and concave in its arguments and has constant returns to scale.

Workers are risk neutral and live infinitely. At any moment in time,each individual can either be employed or unemployed. We assume that each filled job can be destroyed at an exogenous rate δ, which means that an employed worker loses his/her job at rate δ. We denote by r the real interest rate.

1. We assume that the unemployment benefit b is financed by a tax τ on the firms. This means that when a firm hires a worker, its instantaneous profit is equal to: $y - w - \tau$, where y is a worker's productivity in the firm. The fiscal policy is such that the unemployment benefits are kept constant and the budget adjustment is realized through a decrease or increase in taxes τ. Therefore, the government's budget constraint can be written as:

$$\tau = \left(\frac{u}{1-u}\right) b.$$

   When a job is vacant, the firm has has a cost of c to keep its vacancy. By using the Bellman equation, write the steady-state intertemporal utility of a firm holding a vacant job (denoted by V) and that of a firm having a filled job (denoted by J).

2. We assume that the wage w is determined by the Nash bargaining rule, where each worker's bargaining power is denoted by β. Write the Nash maximization program and solve it. Give the exact value of the

negotiated wage w^* as a function of the unemployment rate u. This will be equation (W). For a given θ, how w^* vary with u? Explain.

3. Write the free-entry condition and express each firm's labor demand as a relationship between θ^*, the labor market tightness, and u, the unemployment rate. This will be equation (LD). How θ^* vary with u? Explain.

4. At the steady-state equilibrium of this economy, flows in unemployment must be equal to flows out of unemployment. Write this equation and determine the unemployment rate u, which is equal to the unemployment level U. This will be equation (U). Give a formal definition of the steady-state equilibrium of this economy.

5. By excluding degenerate cases, show that the number of steady-state equilibria is even and equal to either zero or two (multiple equilibria) and thus that the steady-state equilibrium of this economy, if it exists, is never unique.

6. Assume that two steady-state equilibria exist. Denote by $E_H = (\theta_H^*, u_H^*)$ the "good" equilibrium, i.e. the high equilibrium where θ is high, and u and τ are low, and by $E_L = (\theta_L^*, u_L^*)$ the "bad" equilibrium, i.e., the low equilibrium where θ is low, and u and τ are high. Explain the logic and the intuition of the fact that these two equilibria coexist. Give the equation in θ^* that characterizes these two equilibria.

7. Let us analyze the dynamics of this model. Write the dynamic equations of the unemployment rate and the labor market tightness.

8. Determine simultaneously the dynamics of u_t and θ_t. Plot in particular the phase diagram. Show that the steady-state high equilibrium $E_H = (\theta_H^*, u_H^*)$ is a saddle point and steady-state low equilibrium $E_L = (\theta_L^*, u_L^*)$ is either a node or a spiral. Give the intuition of the dynamics of these equilibria.

9. One can Pareto-rank these two equilibria by using the lifetime expected utility of each agent (worker and firm) as a criterion. Can you show that the equilibrium $E_H = (\theta_H^*, u_H^*)$ Pareto-dominates $E_L = (\theta_L^*, u_L^*)$.

10. Assume that the government fiscal policy is now as follows. Taxes are set to the level consistent with the high equilibrium and unemployment benefits are adjusted to balance the budget. This means that, for a constant value of τ, the unemployment benefit level that balances the budget is:

$$b = \frac{1-u}{u}\tau.$$

Show that, in this case, if $\tau < y$, there exists a unique high equilibrium $E_H = (\theta_H^*, u_H^*)$. Show also that this equilibrium is a saddle-point.