A Tale of Two Market Structure: Adverse Selection, Search, and Strategic Venue Selection between Exchange and OTC Markets*

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Abstract

A large number of financial assets are traded in both exchanges and over-the-counter markets (i.e., centralized and decentralized markets, CM and DM thereafter). Moreover, the last century has witnessed the secular migration of asset trading from CM to DM. To this end, this paper develops a tractable model on strategic selection of venue trading to study the causes and consequences of endogenous coexistence of CM and DM. We show that as matching efficiency in DM increases and the information cost decreases, more trade migrates from CM with adverse selection to DM with search frictions. This reveals that investigating implications of adverse selection requires considering the endogenous market structure. Moreover, we find that reducing informational asymmetry with costly information acquisition may be detrimental to welfare. We then apply the model to evaluate the heterogeneous treatment effect of government asset purchase program initiated after the global financial crisis.

Keywords: Exchange vs OTC, Fragmented Financial Markets, Adverse Selection, Search Frictions, Screening

JEL Codes: D61, D82, D83, L10

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1 Introduction

In United States and many other developed countries, a large number of financial assets, such as derivatives, futures, swaps, loan resale, corporate bonds and equity, are traded in both exchanges and over-the-counter markets (i.e., centralized and decentralized markets).[1] Moreover, Biais and Green (2007) document the secular migration of corporate-bond trading from centralized to decentralized markets in the past century. Relevantly, as shown by Harris (2003), equity trading has become less centralized as well. On the one hand, centralized markets (CM), for example the New York stock exchange, have terms of trade publicly displayed, and dispense with search frictions by nature. On the other hand, decentralized markets (DM), i.e., over-the-counter (OTC) markets, are mainly characterized by search and bilateral bargaining.[2] Motivated by the coexistence of frictional markets with different structures, we naturally raise the following questions: (i) Since trading parties could enjoy a publicly displayed price without search frictions in CM, why do some agents bother to trade in DM? (ii) When could CM and DM coexist for asset trading? (iii) What is the implication of market coexistence for asset liquidity?[3] (iv) Does the recent attempts to reduce opacity in markets that suffer from information asymmetry necessarily increase social welfare?

To this end, I propose a tractable model of strategic selection of venue trading to study why lots of financial assets traded in both CM and DM. I characterize CM as competitive markets with adverse selection and DM as decentralized markets with bilateral trading and search and matching frictions. We show that as matching efficiency in DM increases and the information cost decreases, more trade migrates from CM with adverse selection to DM with search frictions. This is consistent with the secular migration of asset trading from CM to DM documented by Biais and Green (2007). In addition to answer above positive and normative questions, in particular under what conditions CM and DM coexist for asset trading, this paper also adds value to the literature in the following ways. First we illustrate a novel interaction of information frictions and search frictions and their roles in explaining endogenous

[1] Quantitatively speaking, nowadays equities are largely traded on exchanges while US Treasuries are mostly traded in over-the-counter markets. Besides, see Hasbrouck (2007) and O'hara (1995) for a comprehensive survey of market structure, and Harris (2003) for the details of the coexistence of exchange and OTC markets.

[2] See Duffie, Garleanu and Pedersen (2005), Lagos and Rocheteau (2009), Hugonnier, Lester and Weill (2014), Atkeson, Eisfeldt and Weill (2015) and Zhang (2017) among others for the analysis of OTC markets. See Duffie (2012) for a survey.

[3] Market liquidity, as emphasized by Chang (2017), consists of two-dimensional measurement. One is on the price while the other one is on the trading speed for trading assets in secondary market. Our paper takes into account both kind of indicators.

market coexistence. Secondly, by characterizing determinants of the migration of asset trading, the model lends insight into the heterogeneous welfare effect of a government asset purchase program. More intriguingly, the endogenous coexistence of CM and DM in our paper suggests that investigating implications of adverse selection requires considering the endogenous market structure. Finally, and surprisingly, we show that reducing informational asymmetry with costly information acquisition may be detrimental to welfare.

As emphasized by Levine (2005), liquid provision and resources reallocation in secondary market is one of the key functions of financial industry. Asset owners could enjoy market liquidity by transferring claims if secondary market functions well. However, adverse selection may dampen potential trade. To address information frictions, we introduce both adverse selection and costly state verification into our model. That is, in addition to posting a pooling price in CM, buyers could also choose to acquire costly information on asset payoffs. Then they can propose an optimal contract with bilateral trading in DM.

In the benchmark model with no information asymmetry on asset payoffs, CM is shown to always dominate DM for asset trading in equilibrium. When asset payoffs and liquidity shock (discount factor, or called trading motive) are private information, there also exists a self-fulfilling equilibrium in which trading parties concentrate in CM.[4] A more intriguing case is whether the equilibrium with market coexistence can be supported. On the one hand, the seller's liquidity shock is always private information. One the other hand, buyers could always stay uninformed about asset payoffs. Then buyers could post a publicly displayed price in CM, at which demand equals supply. Alternatively, buyers can acquire costly information to avoid adverse selection in CM. The informed buyers could then propose a trading menu different from the unique price posted by uninformed buyers in CM. When sellers with high-quality assets self-select into the contracts offered by informed buyers, search frictions may emerge due to coordination failure. That is, information investment and search frictions are two aspects in DM.

Due to strategic complementarity between sellers and buyers on the choice of trading venues, there always exists an equilibrium in which only CM survives for asset exchange. Market coexistence is shown to be sustainable only when adverse selection in CM is severe, matching efficiency in DM is high, and the information cost is low enough. That is, we have multiple equilibria in latter case. To ease the analysis of comparative statics, we always pick up the

[4] In Section 2, we fully characterize three alternative cases, in which either of them is private information and both of them are private information.

equilibrium with markets coexistence whenever it can be supported. Then we conclude that, as matching efficiency in DM increases and the information cost decreases, more trade migrates from CM with adverse selection to DM with search frictions. In the limit, DM with search frictions converges to CM with complete information.

Information investment serves as buyer's natural response to alleviate adverse selection. However, if we aggregate the revenues by all sellers, then immediately we know that information acquisition is purely a resource waste in our exchange economy.[5] In addition to information investment, the unmatched trading in DM also contributes to the deadweight loss. However, since sellers are heterogeneous in their asset payoff and trading motives, closing DM is not Pareto improvement. Based on this observation, we move on to address the heterogeneous effect of a government asset purchase program, for example Troubled Asset Relief Program (TARP). We are particularly interested in the following question. If government is assumed to have access to a lower information cost, or a more efficient matching technology, will all sellers be better off with government intervention? We show that, when government steps in with a self-financing scheme, sellers with high-quality assets are better off while the others are worse off. Therefore, even though in our simple exchange model, which is free of incentive effect in production, a self-financing government asset purchase program does not necessarily makes everyone better off.

Literature review: Our paper focus on the endogenous coexistence of centralized and decentralized market. Pagnotta and Phillipon (2011) also explore market coexistence, but they are engaged in the market fragmentation on trading in organized exchanges with different trading speed. For theory, see Hall and Rust (2003), Miao (2006) and Bolton, Santos and Scheinkman (2016) among others. For empirics, Biais and Green (2007) document the secular migration of corporate-bond trading from CM to DM in the past century. Moreover, as shown in Harris (2003), equity trading has also recently become less centralized. In addition to the literature on market coexistence, our paper is also related to the literature on the liquidity effect of information frictions. Earlier theory include Glosten and Milgrom (1985), Kyle (1985) and Williamson and Wright (1994) among others. Recent burgeoning literature mainly consists of Bolton, Santos and Scheinkman (2011), Kim (2012), Lester, Postlewaite and Wright (2012), Malherbe (2014), Tirole (2012) for theoretical analysis and Eisfeldt (2004), Kurlat (2013), Bigio (2015) and Benhabib, Dong and Wang (2015) for the discussion over business cycles. All of

[5]Buyers are assumed to be fully competitive and thus their gain is irrelevant for calculating the social welfare.

these papers assume a unique price in a competitive centralized market with adverse selection. Moreover, Guerreri, Shimer and Wright (2010), Guerrieri and Shimer (2014a,b), Chang (2017), Chiu and Koeppl (2016) address the effect of information asymmetry on asset trading with search frictions. Alternatively, Tirole and Farhi (2015) and Andolfatto, Berentsen and Waller (2013) adopt costly information acquisition to address the information asymmetry between a single seller and a single buyer. Similar to classic literature on security design, these two papers suggests information investment could be undue diligence under certain conditions. Security design is a burgeoning field with lots of interesting papers, say, DeMarzo and Duffie (1999), DeMarzo (2005) and Dang, Gorton and Holmström (2010) etc. Since our focus is not on security design, we would not give this field a fair treatment for the literature review. Instead, we only focus on information investment, which is also a key issue in this field.

Our paper is related to Guerrieri and Shimer (2014b) and Chang (2017). All the three papers consider two-dimensional private information for asset trading, one is asset payoff while the other is trading motive. See Guerrieri and Shimer (2014b) for the comparison between their paper and Chang (2017). Guerrieri and Shimer (2014b) show when only asset quality is private information, there exists a unique separating equilibrium. Market illiquidity serves as the separating device. However, when both asset quality and the desire to sell are private information, the economy would be characterized by a unique partial pooling equilibrium. Chang (2017), which also considers private information on asset quality and trading motives with the framework of directed search, delivers similar conclusions. There are several key differences. First of all, our paper adopt different modeling strategy to address private information. Guerrieri and Shimer (2014b) use market illiquidity as a signaling device while we adopt costly information acquisition. In our model, CM is subject to adverse selection due to private information on asset's common and private values. Meanwhile, buyers could reduce information asymmetry from two to one dimension and then launch optimal contract to sellers self-selecting into DM. Secondly, the sub-markets with competitive search share very similar market structure in their papers. In contrast, our model offers a framework with endogenous coexistence of CM and DM, two kinds of markets with quite different characteristics. Another related paper is Lester, Shourideh, Venkateswaran and Zetlin-Jones (2015, LSVZ thereafter). Both LSVZ (2015) and my paper analyze the positive and normative implications of screening and adverse selection in frictional markets. Moreover, both of us emphasize that addressing the effects of adverse selection calls for controlling for market structure. The main difference is that LSVZ (2015) demonstrates the intriguing interaction between adverse selection, screening, and imperfect competition, while my paper focuses on the endogenous coexistence of exchange and OTC markets.

The rest of this paper proceeds as follows. Section 2 sets up a stylized model and analyzes agent's choice of trading venues between CM and DM in partial equilibrium. Section 3 closes the model in general equilibrium. Section 4 apply the model to examine the heterogeneous effect of government asset purchase program. Section 5 concludes. All the proofs are put in the Appendix.

2 Model

2.1 Environment

The economy is populated by two kinds of risk-neutral agents and lasts for two periods. First, there is unit measure of asset sellers. Each of them is endowed with one unit of indivisible Lucas tree at the beginning of $t = 1$. Seller's utility function is $U^S(x, \delta) = c_1 + \delta c_2$, where $c_1, c_2 \geq 0$ denotes consumption at $t = 1$ and $t = 2$, and x and δ the idiosyncratic asset payoff and discount factor respectively. Therefore sellers are heterogeneous in both common value and private value. For notational ease, we label them as seller-(x, δ). For simplicity, we assume these two distributions are independent of each other. On one hand, asset payoff is drawn from a continuous distribution $F(x)$ with support $[x_L, x_H]$. Discount factor, on the other hand, conforms to a distribution $G(\delta)$ with support $[0, 1]$. Trees only deliver consumption goods at $t = 2$. We assume sellers cannot produce. Therefore maturity mismatch may emerge if some sellers want to sell their trees to consume in $t = 1$. This is in particular true for sellers with $\delta = 0$.

Secondly, we assume there is a continuum of asset buyers. For simplicity, we assume no occupational choice between buyers and sellers.[6] Buyers have access to a linear production technology with labor input at $t = 1$. There is no aggregate shock to this economy. However, we assume it is not feasible for sellers to issue contingent claims. Besides, no credit is assumed to be enforceable. Additionally, the limited commitment makes it impossible for sellers to signal in secondary market. Consequently, assets serve as medium of exchange, *i.e.*, sellers could transfer asset ownership to buyers for consumption at $t = 1$. In turn buyers would have to produce consumption goods to purchase the trees at $t = 1$, and consume the fruits

[6]Bolton et al (2011b) discusses the endogenous choice between financial service and real business.

at $t = 2$. Therefore when talking about liquidity, we exclusively mean market liquidity rather than funding liquidity.[7]

In addition to staying uninformed, buyers can also acquire costly information. More specifically, buyers could pay information cost κ with their labor disutility. Then they could perfectly detect the payoff of any asset. The action of information investment is publicly observed. Each information investment can only verify the quality of one-unit asset. We denote buyer's utility function as $U^B = -l_1^B - \kappa \cdot \mathbf{1}_{\{\text{Info-invest}\}} + \mathbb{E}\left(c_2^B\right)$, denotes the disutility from producing l_1^B units of goods, and c_2^B denotes the consumption goods at $t = 2$ from the trees the buyer purchase at $t = 1$.[8] Buyers are fully competitive so that they would earn zero profit from asset trading in equilibrium.

To fully characterize the expected revenue $\mathbb{E}\left(c_2^B\right)$, we need to specify the details on how assets are traded between sellers and buyers. On one hand, if certain buyer does not incur information cost, she would have no idea on the exact quality of assets. Thus she can only buy asset with a publicly displayed price p which demand equals supply in equilibrium. On the other hand, if some buyer acquires costly information, she could follow uninformed buyers to post a publicly displayed price p. Alternatively, she could propose a trading menu for sellers self-selecting into the contract. Notice that informed buyers could detect the asset payoff x, but they still cannot directly observe δ, the discount factor of asset sellers. Without loss of generality, informed buyers uses direct mechanism $\{q(x, \delta), \tau(x, \delta)\}$. When sellers with asset payoff x report their type as δ, $q(x, \delta)$ is the probability that an asset transferred to buyers while $\tau(x, \delta)$ is the consumption paid to sellers.

Now it is time for us to be clear on our definition of centralized and decentralized markets (CM/DM). The former is a market where assets are traded at a publicly displayed price p. That is, sellers could always successfully sell their assets in CM at p without any search frictions. In contrast, as noted by Duffie (2012), DM is characterized by search and matching. That is, it takes time for sellers and buyers to find their trading partners. Since we assume each information investment can *only* verify the quality of one unit of asset, sellers and informed buyers will take bilateral trading. Therefore DM emerges in the bilateral trading since sellers

[7] See Brunnermeier and Pedersen (2009) for the details on market and funding liquidity.

[8] Some asset trading is dealer-intermediated in our real life, with corporate bonds just being a case. We assume away the intermediation in this paper. It contributes to great tractability for our focus on equilibrium choice. Or, a cheap interpretation is that we combine the roles of dealers and buyers and is exclusively engaged in the trading frictions due to private information on heterogeneity of seller side. The price of assuming way dealers in DM is that there is no room to use our model to discuss the bid-ask spread and other important dealer-related financial phenomenon.

may fail to coordinate with each other about which buyers to resort to. It is true whether it be random search or competitive (directed) search. We use random search in the baseline model and make robust check with competitive search in the Appendix B. We assume matching technology $m(b,s)$ in DM is exogenously given, increases with both augments, is homogeneous of degree one, $m(b,0) = m(0,s) = 0$, and $m(b,s) \leq \min(b,s)$, where b and s denote the measure of buyers and sellers in DM.

Buyers and sellers move simultaneously. Buyers make their choice on information investment or not. Sellers decides whether and where to trade. For sellers who choose to go to DM but are not successfully matched, we assume they can no longer try CM and instead go directly to next period.[9] In our benchmark, we model liquidity shock in the simplest way as by Diamond and Dybvig (1983). We assume δ conforms to binomial distribution with $\delta \in \{0,1\}$, $\Pr\{\delta = 0\} = \pi$, and $\Pr\{\delta = 1\} = 1-\pi$. In the end, we use Figure 1 to summarize the time-line.

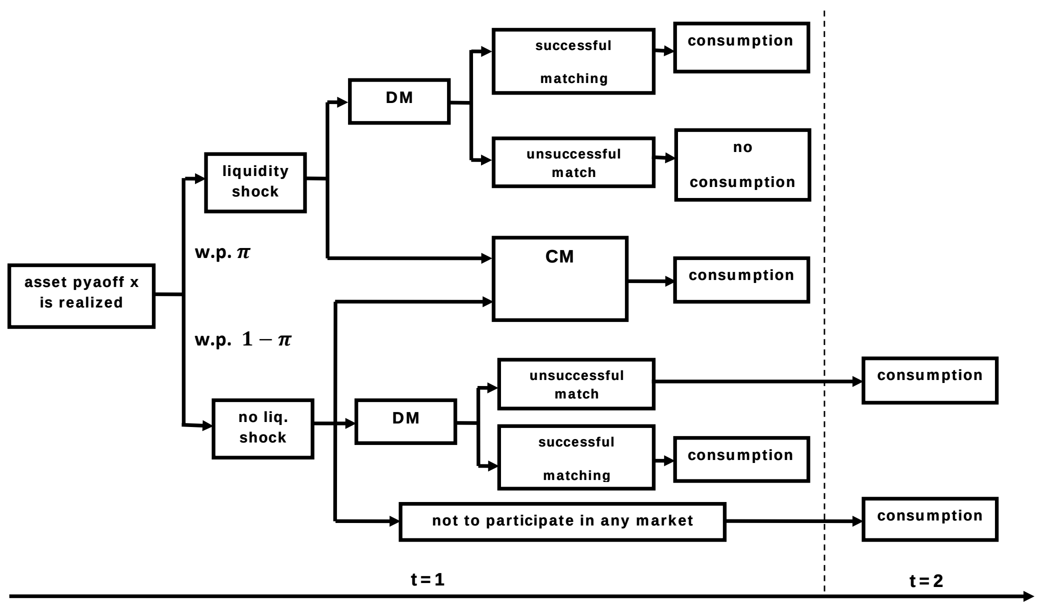

Figure 1: **Raw Time-line with All Possible Paths**

[9]We also consider an alternative scenario in which DM and CM are connected. That is, sellers have no commitment and are allowed to put their order at both markets. If sellers are not matched in DM, they still have the opportunity to liquidate their assets in CM if want to. Most of the qualitative conclusions in the context still hold.

2.2 Seller's Problem

As suggested above, buyers can incur fixed evaluation cost κ and then choose to go to the DM for asset trading. Then buyers would have no information disadvantage on x in DM. The terms of trade in DM for seller-$(x, \delta = 0)$ is then determined by Nash bargaining under complete information. By complete information, we mean both asset payoff x and liquidity shock δ are publicly observable without any cost. It is worth noting that seller's outside option crucially depends on δ. Since we assume sellers cannot trade in CM and DM in the same time, even though two markets coexist, the outside option of seller-(x, δ) going to DM is $x \cdot \mathbf{1}_{\{\delta=1\}}$.

For those with $\delta = 0$, the terms of trade is determined by max $\left\{\omega^{\eta} \cdot (x - \omega)^{1-\eta}\right\}$, which delivers $\omega(x) = \eta x$ for all x. When bargaining with buyers in DM, seller-$(x, \delta = 0)$ and seller-$(x, \delta = 1)$ have different outside option. The outside option is zero and x for the former and latter respectively. Thus strictly speaking, the bargaining setting max $\left\{\omega^{\eta} \cdot (x - \omega)^{1-\eta}\right\}$ is a reasonable for the former but not for the latter group of asset sellers. Fortunately, this subtle observation does not overthrow our analysis to come. Even though the "bargaining power" of seller with $\delta = 1$ could be higher than that of seller with $\delta = 0$, they would never try DM. This claim is immediately obtained by the following argument. Buyers in DM would charge at least something from the trading surplus. As a result, the best possible terms of trade for seller-$(x, \delta = 1)$ would always be strictly lower than x. For those sellers, they could always gain x by waiting until $t = 2$. Thus they would never try DM, even though the rule of splitting trading surplus for them is different from that for sellers with $\delta = 0$. Consequently, even though liquidity shock is always unobservable, buyers can infer it from seller's choice of trading venues.

For those with $\delta = 1$, there is no trading surplus in DM. As a result, although buyers cannot directly detect δ, they could infer that only sellers with $\delta = 0$ would show up in DM. Without loss of generality, we assume buyers always propose the contract as $\{q(x, \delta) = 1, \tau(x, \delta) = \eta x\}$ after paying information cost κ. Then the objective function of seller-(x, δ) is formulated as below.

$$U^S(x, \delta) = \max_{\{\text{CM, DM, Delay}\}} \{c_1 + \delta c_2\}$$

where

$$
\begin{aligned}
a &= \begin{cases} 1, & \text{sell the assets at } t = 1 \\ 0, & \text{keep it until } t = 2 \end{cases} \\
c_1 &= \underset{\{\text{DM, CM}\}}{max} \left\{ \frac{m(b,s)}{s} \cdot \eta x,\ p(x) \right\} \cdot a \\
c_2 &= a \cdot \mathbf{1}_{\{\frac{m(b,s)}{s} \cdot \eta x > p(x)\}} \cdot \left[1 - \frac{m(b,s)}{s} \right] \cdot x + (1-a) \cdot x
\end{aligned}
$$

and b, s denote the measure of buyers and that of sellers in DM respectively, and $p(x)$ the price of asset-x in CM. Notice that we employ random search in the benchmark. Thus (b, s) does not differentiate the measure of trading parties in certain sub-markets. For sellers-$(x, \delta = 0)$, they have to sell their asset at $t = 1$. Thus their discrete choice is reduced to $\max\limits_{\{\text{CM, DM, Delay}\}} \left\{ \frac{m(b,s)}{s} \eta x, p(x) \right\}$. For sellers-$(x, \delta = 1)$, in addition to participating in either DM or CM at either $t = 1$, they could also exercise the option of waiting until the dividend is delivered at $t = 2$. Thus their target is formulated as $\max\limits_{\{\text{CM, DM, Delay}\}} \left\{ \frac{m(b,s)}{s} \eta x + \left(1 - \frac{m(b,s)}{s}\right) x, p(x), x \right\}$.

2.3 Choice of Trading Venues

This part analyzes the choice of trading venues under both complete and incomplete information. For complete information, we mean both asset payoffs x and liquidity shock δ, *i.e.*, trading motives, are publicly observable.

Complete Information

Denote $p(x)$ as the price of asset-x in CM. Since buyers are fully competitive, buyer's profit $x - p(x)$ from buying asset-x should be zero. As a result, $p(x) = x$ for all x with complete information. Then we reach the following proposition.

Proposition 1 ***(Market Participation under Complete Information)*** *When there is no information asymmetry,*

1. *Any seller-$(x, \delta = 0)$ would prefer CM to DM for asset trading at $t = 1$.*

2. *Any seller-$(x, \delta = 1)$ would never try DM. They are indifferent between trading in CM at $t = 1$ and waiting to consume at $t = 2$.*

3. *None of buyers incur information investment. Instead, all of them concentrate in CM in $t = 1$.*

The key message of this proposition is, when there is no information asymmetry on asset payoffs, the CM is preferred to search frictions and bargaining in DM. We move on to the discussion with information asymmetry in the rest of this section.

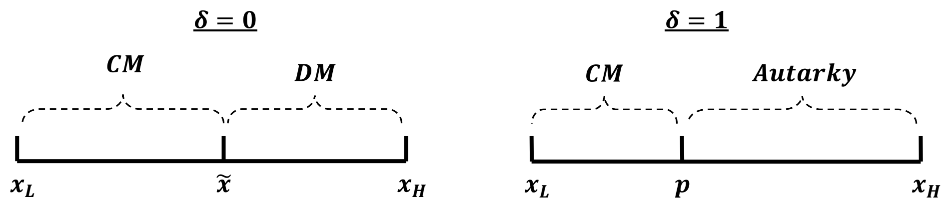

Figure 2: **Choice of trading venues by seller-**(x, δ)

Incomplete Information

When (x, δ) are private information of sellers, $p(x)$ is the same for sellers self-selecting to pool in CM at $t = 1$. Denote $p(x) = p$ in this case. Then we have the following result on choice of trading venues in partial equilibrium.

Proposition 2 ***(Market Participation under Two-dimensional Information Asymmetry)*** *When* both *asset payoff and liquidity shock are seller's private information, market participation is a choice function of seller-(x, δ) from $X \times \Delta = [x_L, x_H] \times \{0, 1\}$ to $\{CM,\ DM,\ Autarky\}$ such that,*

1. **Proposition 3** *(a) For sellers with $\delta = 0$, there exists a cut-off point $\widetilde{x} \in [x_L, x_H]$ such that if $x \geq \widetilde{x}$, they would self-select into DM, and enter CM otherwise at $t = 1$.*

   (b) For sellers with $\delta = 1$, if $x < p$, they would choose CM, and if $x \geq p$, they would participate in neither DM nor CM at $t = 1$, but instead wait to consume at $t = 2$.

   (c) Given $(\widetilde{x}, p)$, the utility function of seller-(x, δ) is refined as below.

$$U^s(x, \delta) = max \left\{ \frac{x}{p + (\widetilde{x} - p) \cdot \mathbf{1}_{\{\delta = 0\}}},\ 1 \right\} \cdot p$$

The main message of this proposition is, when sellers are subject to preference shock, those with high-quality assets tend to sell at DM while the others pool into CM. However, for those without preference shock, since they have the outside option as waiting and consuming by themselves, they would never try DM with search and bargaining. Meanwhile, they would take advantage of CM if their asset's quality is low. For illustration concern, I summarize the choice of trading venues by seller-(x, δ) and the associated gain $U^s(x, \delta)$ in Figures 2 and 3 respectively. Figures 2 and 3 jointly imply that the measure of sellers in DM is

$$s = \pi \cdot [1 - F(\widetilde{x})], \tag{1}$$

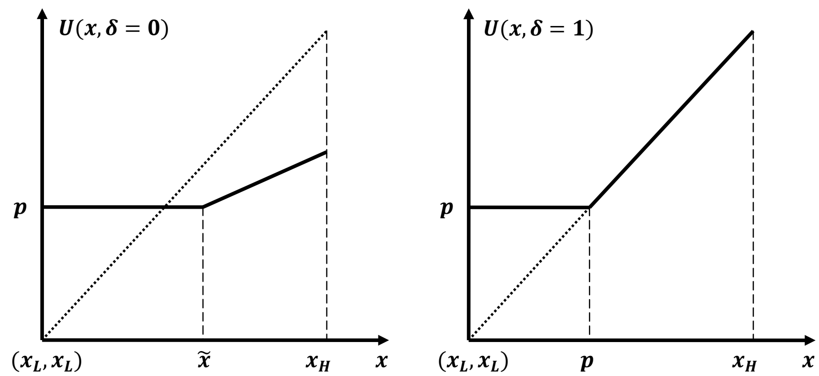

Figure 3: **Expected gain of seller-**(x, δ)

Finally, based on Proposition 2, we show the results on market participation when either x or δ is private information.

Corollary 1 ***(Market participation with only one-dimensional information asymmetry)*** *When either asset payoff and liquidity shock is seller's private information, market participation of sellers is as below.*

1. ***(When only trading motive is private information)*** *In this case, the result is the same as that with complete information on (x, δ) in Proposition 1. That is,*

   (a) *Any seller-$(x, \delta = 0)$ would always prefer CM to DM for asset trading at $t = 1$.*

   (b) *Any seller-$(x, \delta = 1)$ would never try DM. They are indifferent between trading in CM at $t = 1$ and waiting to consume at $t = 2$.*

   (c) *None of buyers incur information investment. Instead, all of them concentrate in CM in $t = 1$.*

2. ***(When only asset payoff is private information)*** *In this case, the result is very similar to the that of Proposition 2, but with $\pi = 1$.*

   (a) *For sellers with $\delta = 0$, there exists a cut-off point $\widetilde{x}^* \in [x_L, x_H]$ such that if $x \geq \widetilde{x}^*$, they would self-select into DM, and enter CM otherwise at $t = 1$.*

   (b) *For sellers with $\delta = 1$, if $x < p$, they would choose CM, and if $x \geq p$, they would participate in neither DM or CM at $t = 1$, but instead wait to consume at $t = 2$.*

(c) *Buyers who are posting price p in CM would never accept sellers with $\delta = 1$. Thus the equilibrium result would perform as if all sellers transferring their assets are $\delta = 0$, i.e., $\pi \equiv Pr(\delta = 0) = 1$.*

That is, if only δ or x serves as private information, seller's choice of trading venues is reduced to the case with complete information and that with two-dimensional informational asymmetry in Proposition 1 and 2 respectively. As a result, this corollary justifies why we stick to the general case with both x and δ as being seller's private information when departing from the case with complete information. Moreover, we would rely on this corollary to simply our argument in Section 4 on welfare analysis by assuming only asset payoff is private information.

2.4 Asset Price in Centralized Market

There are two key variables in our partial-equilibrium analysis. One is p, the price in CM while the other one is $\widetilde{x}$, the cut-off point of choice between CM and DM. We use this section and the next one to reach two equations to determine $(p, \widetilde{x})$. Since buyers are assumed to be competitive in CM, none of them make positive profits in equilibrium. Thus the price in CM is determined as below.

$$p = \frac{\pi F(\widetilde{x})\mathbb{E}(x|x \leq \widetilde{x}) + (1-\pi)F(p)\mathbb{E}(x|x \leq p)}{\pi F(\widetilde{x}) + (1-\pi)F(p)} \tag{2}$$

The LHS of Eq. (2) is buyer's cost for one unit of asset. The RHS is the average value of assets pooling in CM. Buyers are uninformed of the true value of each asset in CM. However, as implied in the above proposition, buyers have rational expectation of $F^{CM}(x)$, the true (truncated) distribution of asset payoffs in CM. One source is from those sellers with $(x \leq \widetilde{x}, \delta = 0)$ while the other one from those with $(x \leq p, \delta = 1)$. The numerator and the denominator of Eq. (2) are the total value and total measure of assets in CM respectively.

Lemma 1 ***(Asset Price in CM)*** *Given any $(\widetilde{x}, \pi)$, Eq. (1) has a unique solution as $p = P_{AS}(\widetilde{x}, \pi)$.*

1. *For the general case, we have*

   (a) $\partial P_{AS}/\partial \widetilde{x} > 0$ *and* $\partial P_{AS}/\partial \pi > 0$.

   (b) $x_L = P_{AS}(\widetilde{x} = x_L, \pi) \leq p \leq P_{AS}(\widetilde{x}, \pi = 1) = \mathbb{E}(x|x \leq \widetilde{x}) \leq min\{\widetilde{x}, \mu\}$.

   (c) $P_{AS}(\widetilde{x}, \pi = 0) = x_L$. *Thus CM completely collapses when* $\pi = 0$.

2. When $x \stackrel{U}{\sim} X = [x_L, x_H]$, we have

$$p = P_{AS}(\widetilde{x}, \pi) = \varphi(\pi) \cdot \widetilde{x} + [1 - \varphi(\pi)] \cdot x_L, \tag{3}$$

where $\varphi(\pi) \equiv \frac{\sqrt{\pi}}{\sqrt{\pi}+1}$ and $Pr\{\delta = 0\} = \pi \in [0, 1]$.

Several comments are made here. First of all, when $\pi = 1$, *i.e.*, all sellers would be hit by liquidity shock at $t = 1$, then $p = P_{AS}(\widetilde{x}, \pi = 1) = \mathbb{E}(x|x \leq \widetilde{x})$, a classic problem on adverse selection. Secondly, when $\pi = 0$, *i.e.*, all sellers pooling in CM is simply due to selling lemons rather than liquidating for liquidity need, then CM simply collapses because of severe adverse selection.

2.5 Free Entry of Information Investment

To make the analysis on market coexistence non-trivial, we have to assume buyer's information investment κ is small relative to the average asset quality. Otherwise, buyers would have no incentive to pay the cost and trade in the DM. In anticipating this scenario, both sellers and buyers would always concentrate in CM. Buyer's free entry condition in DM is then formulated as below.

$$\begin{aligned} \left[\frac{m(b,s)}{b}(1-\eta)\mathbb{E}(x|x \geq \widetilde{x}) - \kappa\right] \cdot b &= 0, \\ \frac{m(b,s)}{b}(1-\eta)\mathbb{E}(x|x \geq \widetilde{x}) - \kappa &\leq 0, \ b \geq 0, \end{aligned}$$

where $s = \pi[1 - F(\widetilde{x})]$. Assume DM exists, *i.e.*, $b > 0$ and $s > 0$, and denote the market tightness as $\alpha \equiv \frac{s}{b}$, then we have

$$m(1, \alpha)(1 - \eta)r(\widetilde{x}) = \kappa. \tag{4}$$

Since $r(\widetilde{x}) \equiv \mathbb{E}(x|x \geq \widetilde{x})$ increases with $\widetilde{x}$ and $m(1, \alpha)$ increases with α, applying Implicit Function Theorem to the above equation immediately suggests a negative relationship between α and $\widetilde{x}$. I denote it as $\alpha = \alpha(\widetilde{x})$, a decreasing function of $\widetilde{x}$. In turn, when $\widetilde{x} \in (x_L, x_H)$, by definition it is characterized by $\frac{m(b,s)}{s}\eta\widetilde{x} = p$, which can be further refined as below.

$$p = P_{FE}(\widetilde{x}) \equiv \underbrace{m\left(\frac{1}{\alpha(\widetilde{x})}, 1\right)}_{ext.\ margin} \cdot \underbrace{\eta\widetilde{x}}_{int.} \tag{5}$$

Since $\alpha(\widetilde{x})$ decreases with $\widetilde{x}$, the LHS of Eq. (3) delivers a positive relationship between $\widetilde{x}$ and p. We denote is as $p = P_{FE}(\widetilde{x})$. It is worth noting that, since $m(1, \alpha) < \lambda$, to guarantee that $\alpha(\widetilde{x})$ always exists, we must have $\mathbb{E}(x|x \geq \widetilde{x}) > \frac{\kappa}{\lambda(1-\eta)}$. A sufficient condition is $\mathbb{E}(x|x \geq$

$x_L) = \mu > \frac{\kappa}{\lambda(1-\eta)}$. If $\mathbb{E}(x|x \geq \widetilde{x}) \leq \frac{\kappa}{\lambda(1-\eta)}$ holds for some $\widetilde{x}$, then we have a corner solution as $b = \alpha(\widetilde{x}) = 0$. An extreme case is that, if $x_H = \mathbb{E}(x|x \geq x_H) \leq \frac{\kappa}{\lambda(1-\eta)}$, then $\alpha(\widetilde{x}) = 0$ for all $x \in X \equiv [x_L, x_H]$. Another comment is, when $\widetilde{x} = x_H$, we have $s = 0$ and thus we always have $b = 0$ in this case.

Lemma 2 ***(Buyer's Free Entry Condition on Information Investment)***

1. *Given any p, there exists a unique $\widetilde{x}$ which satisfies Eq. (3) and is denoted as $\widetilde{x} = X_{FE}(p; \lambda, \kappa, \eta)$. X_{FE} and P_{FE} have the following property.*

$$\partial X_{FE}/\partial p > 0, \partial X_{FE}/\partial \kappa > 0, \partial X_{FE}/\partial \eta < 0, P_{FE}(x_L) < x_L.$$

2. *When $m(b, s) = \lambda \cdot min\{b, s\}$ and $x \overset{U}{\sim} X = [x_L, x_H]$,*

   (a) *Eq. (3) is simplified as*

$$\lambda \cdot min\{1, \alpha\}(1-\eta)r(\widetilde{x}) = \kappa, \tag{6}$$

   where $\alpha \equiv \frac{s}{b}$ and $r(\widetilde{x}) \equiv \mathbb{E}(x|x \geq \widetilde{x}) = \frac{\widetilde{x}+x_H}{2}$.

   (b) *The marginal seller between CM and DM is characterized as*

$$\lambda \cdot min\left\{\frac{1}{\alpha}, 1\right\} \cdot \eta \cdot \widetilde{x} = p. \tag{7}$$

When p increases, terms of trade in CM becomes more favorable for sellers with $\delta = 1$ and thus more of them switch to CM. In the same spirit, when κ increases, given any $\widetilde{x}$, the market tightness would be more tough for sellers and thus some of them would then pool into CM. Moreover, when η increases, the terms of trade in DM looks more attractive and thus sellers in CM would then switch to DM.

In this subsection, we have so far focused on the scenario of market coexistence. That is, we implicitly assume that $\widetilde{x} \in (x_L, x_H)$ and $s, b > 0$. When $\widetilde{x} = x_H$, *i.e.*, only CM exists, then $s = 0$ by Eq. (1). In turn, the free entry condition implies that $b = 0$. In this case, we have

$$\widetilde{x} = X_{FE}(p; \lambda, \kappa, \eta) = x_H, \tag{8}$$

which holds for all $p \geq 0$. However, another polar case with $\widetilde{x} = x_L$ can never be possible. Here is the reasoning. Since the lower bound of asset payoff is $x_L > 0$ and the competitive buyers would offer $p \geq x_L$, for sellers with $x = x_L$, they would always prefer the immediacy of CM to the search and bargaining in CM. By continuity, for sellers with x being close to $x = x_L$ would always choose CM over DM. Therefore it could never be that CM is totally replaced by DM unless $x_L = 0$.

3 Equilibrium Choice of Trading Venues

3.1 General Equilibrium

Combining AS and FE condition solves $(p, \widetilde{x})$, which is illustrated in Figure 4. Then we reach the general equilibrium choice of trading venues as below.

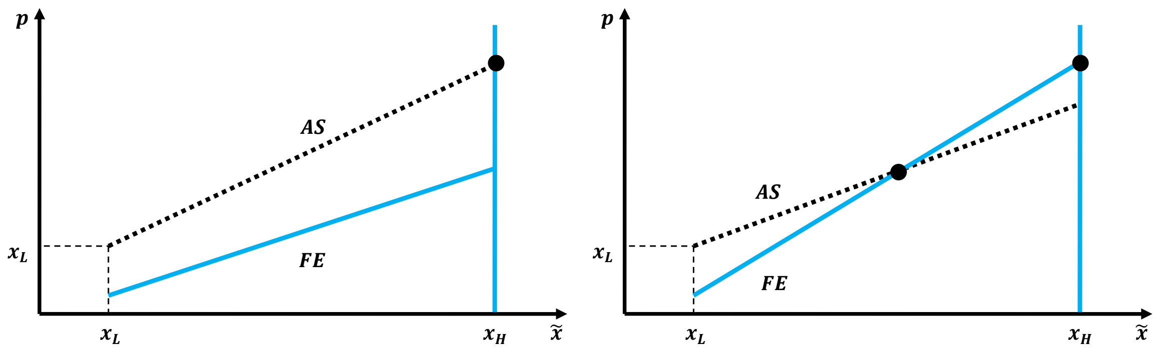

Figure 4: $(p, \widetilde{x})$ **are jointly determined by the intersection AS-curve and FE-curve.**

Definition 1 *The equilibrium consists of (i) p, price in CM, $\widetilde{x}$, the cut-off point of the choice of trading venues, b, the measure of buyers making information investment and entering DM, s, the measure of sellers self-selecting into DM, α, the market tightness of DM at $t = 1$; (ii) seller's choice of trading venues at $t = 1$, such that*

1. *Given $(p, \widetilde{x})$, seller's choice of trading venues is characterized by Figure 3.*

2. *$(p, \widetilde{x})$ are jointly determined by Eq. 3, Eq. (3) and Eq. (8).*

3. *Given p and $\widetilde{x}$, s is given by (1) α by Eq. (4) and b in turn by $b = s/\alpha$.*[10]

We use the following proposition to fully characterize the equilibrium choice of trading venues.

Proposition 4 ***(Equilibrium Choice of Trading Venues)*** *Assume $x \overset{U}{\sim} X = [x_L, x_H]$ and $m(b, s) = \lambda \cdot min\{b, s\}$. Denote $\underline{\kappa} \equiv \frac{\lambda(1-\eta)}{2}\left[(\frac{1-\varphi}{\lambda\eta-\varphi})x_L + x_H\right]$ and $\overline{\kappa} \equiv \lambda(1-\eta)x_H$.*

[10]If $\widetilde{x} = x_H$ in equilibrium, we have $b = s = 0$ and then the market tightness $\alpha = s/b$ is not well-defined, but it would not bother our analysis then.

1. *When $\lambda\eta > \varphi$ and $\sigma > \hat{\sigma} \equiv (\frac{1-\lambda\eta}{1+\lambda\eta-2\varphi})\mu$ (i.e., $\lambda\eta > \varphi + (1-\varphi)\frac{x_L}{x_H}$), where $\varphi = \varphi(\pi) \equiv \frac{\sqrt{\pi}}{\sqrt{\pi}+1}$, we have $\overline{\kappa} > \underline{\kappa}$ and*

$$\widetilde{x} = min\left\{x_H,\, max\left\{(\frac{1-\varphi}{\lambda\eta-\varphi})x_L,\, \frac{2\kappa}{\lambda(1-\eta)} - x_H\right\}\right\} = \begin{cases} x_H & if\ \kappa > \overline{\kappa} \\ \frac{2\kappa}{\lambda(1-\eta)} - x_H & if\ \underline{\kappa} < \kappa \leq \overline{\kappa} \\ (\frac{1-\varphi}{\lambda\eta-\varphi})x_L & if\ \kappa \leq \underline{\kappa} \end{cases}.$$

$\widetilde{x} = x_H$ can be also supported in this case, but it is not stable.

2. *When $\lambda\eta \leq \varphi$ or $\sigma \leq \hat{\sigma}$ (i.e., $\lambda\eta \leq \varphi + (1-\varphi)\frac{x_L}{x_H}$), we have $\widetilde{x} = x_H$ for all $\kappa \in \mathbb{R}_+$.*

We use Figure 5 to illustrate the above proposition. Intuitively, when κ is large enough, *i.e.*, $\kappa > \overline{\kappa}$, even though matching efficiency is high in DM and adverse selection is low in CM, only CM would survive for asset trading. When κ decreases, it is not only more likely that market coexistence could emerge, but also more trading would switch to DM given that coexistence can be sustained. The upper panel of Figure 6 treats other exogenous variables as given and focuses on the effect of information cost κ on equilibrium choice of trading venues.

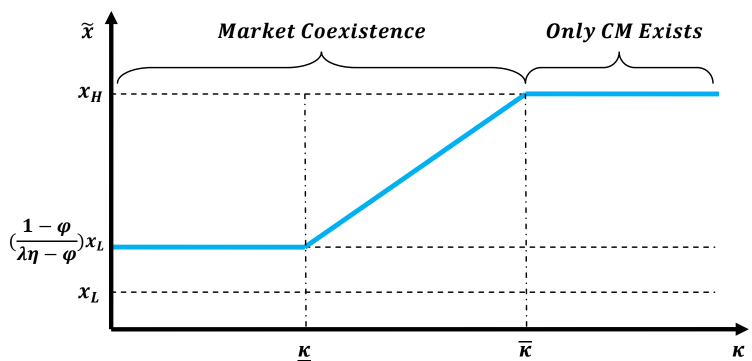

Figure 5: **Equilibrium Choice of Trading Venues** ($\lambda\eta > \varphi$ **and** $\sigma > \hat{\sigma} \equiv (\frac{1-\lambda\eta}{1+\lambda\eta-2\varphi})\mu$).

Based on Figure 5, given any κ, we use the upper panel to of Figure 6 demonstrate the implication of the increase of matching efficiency in DM for choice of trading venues. It is clearly shown that, holding κ constant, both extensive-margin and intensive-margin changes with λ. The second panel suggests that $\widetilde{x}$ increases with π. The intuition is, when π decreases, the adverse selection tends to be more severe in CM and thus DM looks more attractive for high-quality sellers and thus $\widetilde{x}$ decreases. The first three panels illustrates the monotone relationship between $\widetilde{x}$ and (κ, λ, π) respectively. In contrast, the lower panel suggests the relationship between $(\eta, \widetilde{x})$ is not monotone. When seller's bargaining power η increase, which may be due to the increasing competition of buyers in DM, the direct effect is that $\widetilde{x}$ would decreases since the terms of trade in DM looks more attractive. Meanwhile, when η increases, the proportion of what buyers could get from trading in DM would decrease and thus they have less incentive to enter. In turn, seller's matched probability in DM would decreases, which would discourage sellers from choosing DM over CM. That is, $\widetilde{x}$ would increases in response to the second effect. The lower panel implies that the first effect is dominant when information cost κ is low enough while just opposite when κ is high.

3.2 Trading Share and Distribution of Asset Payoff

After obtaining the equilibrium values on $(p, \widetilde{x})$, we obtain the determinants of trading share in both markets. Moreover, we characterize the distribution of asset payoffs in CM and DM.

Trading Share in CM and DM

According to Figure 5, the measure of sellers participating in either CM or DM at $t = 1$ is $\omega = \pi + (1 - \pi)F(p)$. As a result, *conditioning on* trade exercised at $t = 1$ and using Eq. (9), the (truncated) trading share in CM is

$$\rho^{CM} = \frac{\pi F(\widetilde{x}) + (1 - \pi)F(p)}{\pi + (1 - \pi)F(p)} = \frac{\pi \widetilde{x} + (1 - \pi)p - x_L}{\pi x_H + (1 - \pi)p - x_L}, \tag{9}$$

First of all, if $\lambda\eta \leq \varphi$ or $\sigma \leq \hat{\sigma}$, as implied by Proposition 5, we always have $\rho^{CM} = 1$. Secondly, if $\lambda\eta > \varphi(\pi)$, we have

$$\rho^{CM} = \begin{cases} 1 & if\ \kappa > \overline{\kappa} \\ \frac{\frac{2[\pi+\varphi(1-\pi)]}{\lambda(1-\eta)}\kappa - 2\varphi(1-\pi)\mu - 2\pi\mu}{\frac{2\varphi(1-\pi)}{\lambda(1-\eta)}\kappa - 2\varphi(1-\pi)\mu} & if\ \underline{\kappa} < \kappa \leq \overline{\kappa} \\ \frac{[\pi+(1-\pi)\lambda\eta](\frac{1-\varphi}{\lambda\eta-\varphi}) - 1}{\pi(\frac{\mu+\sigma}{\mu-\sigma}) + (1-\pi)\lambda\eta(\frac{1-\varphi}{\lambda\eta-\varphi}) - 1} & if\ \kappa \leq \underline{\kappa} \end{cases}, \tag{10}$$

Therefore, in the presence of market coexistence, we have the following comparative statics:

$$\frac{\partial \rho^{CM}}{\partial \mu} \geq 0,\ \frac{\partial \rho^{CM}}{\partial \sigma} \leq 0,\ \frac{\partial \rho^{CM}}{\partial \kappa} \geq 0,\ \frac{\partial \rho^{CM}}{\partial \lambda} \leq 0,\ \frac{\partial \rho^{CM}}{\partial \eta} \lesseqgtr 0,\ \frac{\partial \rho^{CM}}{\partial \pi} \geq 0.$$

As implied in Eq. (9), the effect of λ, η, etc. on ρ^{CM} is through their impact on $\widetilde{x}$, which in turn works on ρ^{CM}. First of all, when the adverse selection is alleviated, *i.e.*, $\frac{\sigma}{\mu}$ decreases, the trading share in CM increases. Secondly, when λ increases, say, due to IT improvement, the DM tends to be more attractive for sellers and thus the trading share in CM shrinks. The same logic

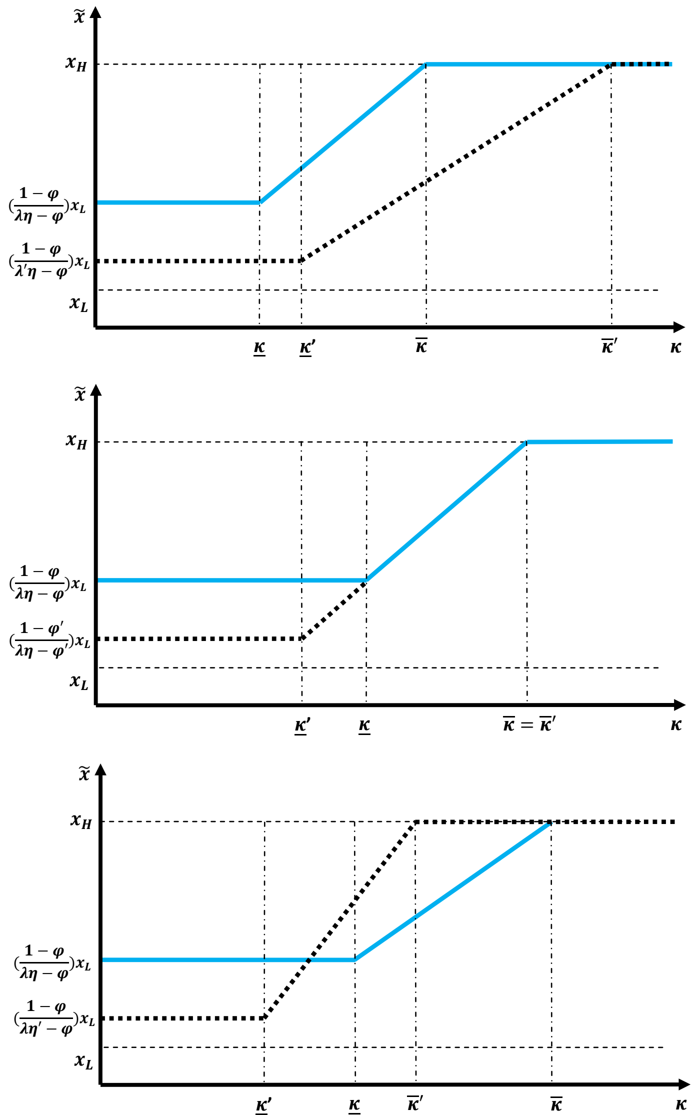

Figure 6: **Top: when λ increases; middle: when π decreases; bottom: when η increases.**

applies to the argument on the effect of π on ρ^{CM}. Thirdly, when π increases, the proportion of sellers with preference shock rather than selling lemons increases. The average quality of assets in CM increases and thus more sellers would trade in CM, which boosts ρ^{CM}. Finally and again, since the information cost κ has no role in neither $\widetilde{x}$ nor p due to the specification on matching function, it does not affect ρ^{CM} provided κ is low enough. In general, when κ decreases, due to financial deregulation or IT improvement, DM tends to absorb more sellers, *i.e.*, $\widetilde{x}$ would decrease and low ρ^{CM}. In sum, the exercise of comparative statics ρ^{CM} lends us insight on the secular migration of bond trading in the past century, which is well documented by Biais and Green (2007). However, it is worth noting that the sign of $\frac{\partial \rho^{CM}}{\partial \eta}$ is ambiguous. Here is the intuition. On one hand, when η increases, the terms of trade in the intensive margin looks more attractive to sellers. On the other hand, the increase of η discourages buyers from making information investment in the extensive margin. In turn, it would be less likely for sellers to be matched with buyers in DM. It is the trade-off between intensive and extensive margin by η that makes ρ^{CM} not monotone with η.

Distribution of Asset Payoffs

First, given market coexistence, the truncated distributions of asset quality in DM and that in CM are given respectively as below.

$$F^{DM}(x) = \begin{cases} 1 & when\, x \in (x_H, +\infty) \\ \frac{F(x)-F(\widetilde{x})}{1-F(\widetilde{x})} & when\, x \in (\widetilde{x}, x_H] \\ 0 & when\, x \in (-\infty, \widetilde{x}] \end{cases} \quad F^{CM}(x) = \begin{cases} 1 & when\, x \in (\widetilde{x}, +\infty) \\ \frac{\pi F(x)+(1-\pi)F(p)}{\pi F(\widetilde{x})+(1-\pi)F(p)} & when\, x \in (p, \widetilde{x}] \\ \frac{F(x)}{\pi F(\widetilde{x})+(1-\pi)F(p)} & when\, x \in (x_L, p] \\ 0 & when\, x \in (-\infty, x_L] \end{cases}, \tag{11}$$

and thus $F^{DM}(x) \leq F(x) \leq F^{CM}(x)$. That is, in the presence of adverse selection, high-quality assets tend to be sold in DM while low-quality ones prefer the immediacy of CM. Bolton, Santos and Scheinkman (2016) document a similar theoretical finding. However, it is worth mentioning that, as argued in the literature review, the information structure differs in our papers and the coexistence of CM and DM is endogenous.

Secondly, when only CM exists for asset trading, $\widetilde{x} = x_H$ and thus $F^{DM}(x)$ is degenerate. $F^{CM}(x)$ is modified as below.

$$F^{CM}(x) = \begin{cases} 1 & when\, x \in (x_H, +\infty) \\ \frac{\pi F(x)+(1-\pi)F(p)}{\pi+(1-\pi)F(p)} & when\, x \in (p, x_H] \\ \frac{F(x)}{\pi+(1-\pi)F(p)} & when\, x \in (x_L, p] \\ 0 & when\, x \in (-\infty, x_L] \end{cases}. \tag{12}$$

3.3 Aggregation with Information Investment

We close this section with a remark on information use. Since our model only considers an exchange economy, the aggregate asset payoffs are fixed. The aggregation with every sellers having the same weight simply suggests that information investment by buyers is a waste of social resources. Following this line of argument, forbidding trade in DM is seemingly socially desirable. Moreover, we can reply on Proposition 3 to obtain the equilibrium values on p, the price in CM, as well as q, the weighted revenue. We illustrate both of them in Figure 7. That is, the emergence of DM with costly information acquisition dampens both the liquidity in CM and the average asset revenues.

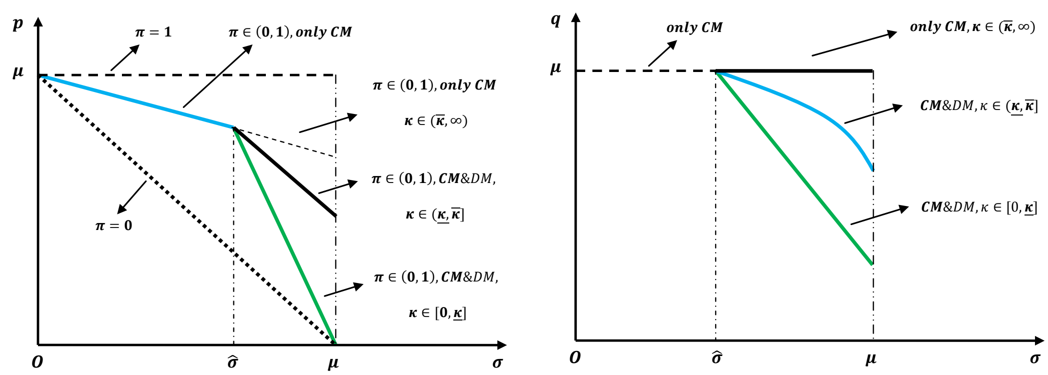

Figure 7: **Price in CM (p) and the Weighted Asset Price (q)**

However, since sellers are ex ante heterogeneous in asset payoffs and liquidity shock, the aforementioned simple weighted calculation is misleading to some extent. As shown in Proposition 3, market coexistence can be sustained under some conditions. In this scenario, sellers with high-quality asset, prefer to bear search friction in DM rather than subsidize low-quality assets in CM. As a result, closing DM would make those sellers worse off. The discussion on the government asset purchase program equips us with a further illustration of this observation.

4 Government Asset Purchase Program

Prior to the financial recession, MBS was considered to be information insensitive assets and thus there did not appear to exist an information asymmetry. However, the outbreak of the financial crisis reminded the market of the potential information asymmetry within the MBS market. Consequently, financial markets tended to be illiquid and some markets, such as the federal funds market, were also frozen. See Heider, Hoerova and Holthausen (2015) and Gale and Yorulmazer (2013) among others for the background description and theoretical explanation.

The US government launched the Troubled Asset Relief Program (TARP) to curb the recent financial crisis. More specifically, the US Treasury implemented the TARP by purchasing mortgage backed securities (MBS) from the financial institutions.[11] In this section, we use the baseline model to address the implication of government intervention for the seller's welfare. We focus on the seller's welfare since buyers are assumed to be fully competitive and thus they would make zero profit in equilibrium. In particular, we raise the following question. Does a self-financing government intervention make all sellers better off? If not, how would the heterogeneous treatment effect be related to the seller's asset quality?

Thanks to Corollary 1, we can concentrate on the simplified case with $\pi = 1$, *i.e.*, all sellers are hit by liquidity mismatch and thus have to sell their assets to buyers before the asset payoffs are realized. Then we can index each seller as seller-x rather than seller-(x, δ) in the baseline. Due to the free entry condition of information investment and trading in the decentralized market (DM), if the government has to incur a higher information cost than do the normal buyers in the baseline, or if its matching efficiency in the DM is lower, then the government would make a loss from its intervention. To make the analysis non-trivial, we assume the asset purchase program is self-financing. In turn we make the following assumption.[12]

Assumption 1 *The government enjoys a lower information cost than buyers, and market coexistence is always sustainable*, *i.e.*, $\kappa_g < \kappa_b < \overline{\kappa} \equiv \lambda(1-\eta)x_H$.

To implement the program, the government issues perfectly enforceable debts to buyers at the beginning of $t = 1$. Thus the government receives consumption goods produced by buyers. When government steps into asset markets, it does not necessarily have an information advantage over the uninformed buyers in the baseline on asset payoffs. We adopt a more reasonable assumption by treating the government in a similar position as uninformed buyers. That is, the government could always set up a pooling price in the CM. Alternatively, the

[11] See the following link for more details of this program: http://www.federalreserve.gov/bankinforeg/tarpinfo.htm.

[12] Alternatively, we could assume $\lambda_g > \lambda_b$, *i.e.*, the government would enjoy a higher matching efficiency in DM after paying the same information cost. Moreover, we can easily relax the assumption that $\kappa_b < \overline{\kappa}$.

government can make an information investment and kick off bilateral trade with sellers in the DM. They can also launch the trade in both markets in the same time.

In sum, with these consumption goods at hand, the government buys seller's assets in the CM, and decides whether or not to pay the information cost and buy assets from DM. At $t = 2$, the government receives consumption goods from the pooling assets it purchases from CM (and DM, if it coexists with CM) at $t = 1$. The government clears its liabilities by repaying buyers with the goods. Since buyers are fully competitive, buyers make zero profit just like the self-financing government intervention does.

On the one hand, since the information cost of government is lower than that of the normal buyers, the free entry condition on information investment in DM suggests that only the government survives in asset exchange in DM with information investment. On the other hand, since the government is self-financing and buyers are fully competitive, neither of them gain positive profit from trading in CM. Without loss of generality, we assume only the government trades with sellers in the CM. Therefore in the presence of Assumption 1, only the government would trade with sellers in either markets in equilibrium. We summarize the key findings in the following proposition.

Proposition 5 ***(Welfare Effect of Government Asset Purchase Program)*** *Under Assumption 1, a self-financing government asset purchase program makes high-quality sellers better off while the low quality sellers worse off. More specifically, there exists a cutoff point $\widehat{x} \in (x_L, x_H)$ such that,*

1. *Sellers with $x \geq \widehat{x}$ are better off. Moreover, the net gain strictly increases with their asset quality x.*

2. *Sellers with $x < \widehat{x}$ are worse off. Moreover, the net loss weakly increases with their asset quality x.*

The above proposition states that, with Assumption 1, *i.e.*, even though the government has an information advantage than the normal buyers, the government cannot deliver a Pareto improvement for the heterogeneous sellers. The decrease of the information cost by the government encourages it to acquire more information in the DM. As a result, sellers who stay in the DM after the government intervention enjoy a more favorable extensive margin. Moreover, the favorable market tightness in general equilibrium drives more sellers to switch from the CM to the DM. Therefore the average quality of assets in the CM decreases. In turn, the pooling price in the CM decreases and those who continue to trade in the CM are worse off. Consequently some sellers are better off while the others are worse off. We illustrate the logic and the cut-off value of the above proposition in Figure 8.

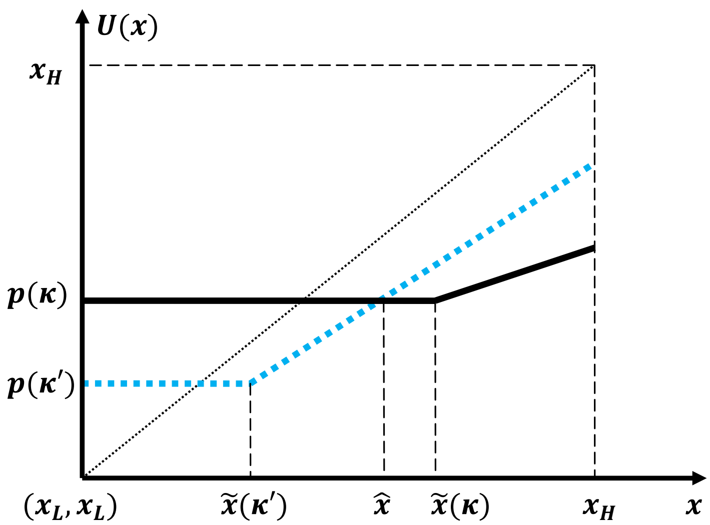

Figure 8: **When government steps in with a lower information cost** ($\kappa' < \kappa$)**.**

5 Conclusion

I develop a tractable model of strategic selection of venue trading to study why lots of financial assets are traded in both exchange and OTC markets (centralized vs decentralized markets, i.e., CM vs DM). There are two-dimensional private information, one is asset payoff while the other one is liquidity shock of asset sellers. On one hand, the latter dimension is always unobservable by others. Buyers can either stay uninformed or choose to acquire costly information on the former dimension. If buyers incur no information cost, then they post a pooling and publicly displayed price at which asset demand equals supply. This is what we mean by centralized market (CM), which is free of search friction, but is subject to adverse selection. In contrast, those buyers with costly information acquisition may propose a trading menu different from the pooling price. Since we assume each information investment can only be used to detect the quality of one unit of asset, the bilateral trading between sellers and informed buyers may be subject to search frictions. This is what we mean by decentralized market (DM), which is characterized with search friction and bilateral bargaining. That is, the endogenous information investment delivers the emergence of DM with bilateral trading. We show that as matching efficiency in DM increases and the information cost decreases, more trade migrates from CM with adverse selection to DM with search frictions. This is consistent with the secular migration of asset trading from CM to DM documented by Biais and Green (2007). Meanwhile, the endogenous coexistence of CM and DM in our paper suggests that investigating implications of adverse selection requires considering the endogenous market structure. Moreover, we show that reducing informational asymmetry with costly information acquisition may be detrimental to welfare.

Due to strategic complementarity between sellers and buyers, there always exists an e-quilibrium in which only CM survives for asset exchange. To ease the analysis comparative statics, we always pick up the equilibrium with markets coexistence whenever it can be supported. Market coexistence emerges only when the following three conditions are satisfied: i) the search friction in DM is low enough, ii) the information friction in CM is severe enough, and iii) the information cost is low enough. Given market coexistence, the trading share of DM over CM increases with matching efficiency in DM and severeness of adverse selection in CM, while decreases with information cost. Then we conclude that, as matching efficiency in DM increases and the information cost decreases, more trade migrates from CM with adverse selection to DM with search frictions. In the limit, DM with search frictions converges to CM with complete information.

Our model with information and search frictions is more than just explaining conditions under which CM and DM coexist for asset trading. We also address the implication of government asset purchase program, such as the Troubled Asset Relief Program (TARP), through the lens of our model. Since sellers are heterogeneous in asset payoffs, even though the government is better at information cost or matching efficiency, the treatment effect of self-financing government asset purchase program is heterogeneous. Moreover, in the presence of government intervention, we show that sellers with high-quality assets are better off while the others are worse off in general equilibrium. Therefore even though the government has an information advantage than the normal buyers, the government cannot generate a Pareto improvement for all those heterogeneous sellers.

We close the paper with several possible lines for future research. First, we assume assets are indivisible. This assumption is innocuous in the paper since we also assume all agents are risk neutral. Both restrictions contribute greatly to tractability. It could be interesting to extend the idea into the scenario with perfect divisible asset. The advancement from indivisibility to divisibility is not a trivial exercise. As emphasized by Lagos and Rocheteau (2009), "...As a result of the restrictions they imposed on asset holdings, existing search-based theories neglect a critical feature of illiquid markets, namely, that market participants can mitigate trading frictions by adjusting their asset positions to reduce their trading needs...". Both Lagos and Wright (2005) and Lagos and Rocheteau (2009) have contributed a tractable framework for asset trading with perfectly divisible asset. Secondly, to neatly model endogenous information acquisition and the emergence of DM, we assume direct trading between sellers and buyers in a finite-horizon model. In our real life, however, a large number of asset trading in DM are dealer-intermediated, say corporate bonds. To better characterize the trading details in DM, such as bid-ask spread, it may be worthwhile for us to introduce dealer between sellers and buyers in DM. Thirdly, it might be desirable for us to integrate the idea in this paper into a dynamic general equilibrium model. Eisfeldt (2004) and Kurlat (2012), among others, are excellent examples of integrating pooling price with adverse selection into RBC models. As suggested throughout this paper, buyers in our paper undertake endogenous level of information investment to lessen adverse selection. Furthermore, we have endogenous trading venues for market liquidity. In sum, the RBC model with our story might deliver additional insights for dynamic decision on real investment and information investment and their interactions with each other.

Appendix

A Proofs

A. 0.1 Proof of Proposition 1 and 2:

Substituting c_1 and c_2 into the objective function yields

$$\begin{aligned} U^S(x,\delta) &= \max_{a\in\{0,1\}}\{max\{\frac{m(b,s)}{s}\eta x,\ p(x)\}\cdot a+\delta\cdot[a\cdot\mathbf{1}_{\{\frac{m(b,s)}{s}\cdot\eta x>p(x)\}}\cdot(1-\frac{m(b,s)}{s})\cdot x+(1-a)\cdot x]\} \\ &= \begin{cases} \max_{a\in\{0,1\}}\{max\{\frac{m(b,s)}{s}\eta x,\ p(x)\}\cdot a\} & when\ \delta=0 \\ \max_{a\in\{0,1\}}\{max\{\frac{m(b,s)}{s}\eta x,\ p(x)\}\cdot a+a\cdot\mathbf{1}_{\{\frac{m(b,s)}{s}\cdot\eta x>p(x)\}}\cdot(1-\frac{m(b,s)}{s})\cdot x+(1-a)\cdot x\} & when\ \delta=1 \end{cases} \end{aligned}$$

As a result, when $\delta=0$, $a^*=1$, *i.e.*, investors with preference shock have to sell the claim of their projects. Investors with $\delta=1$, however, could either participate in centralized or decentralized market ($a=1$) or simply wait till $t=2$ ($a=0$). However, the above optimization implies that investors would never try centralized market due to search friction and bargaining.

First of all, competitive buyers set $p(x)=x$ in complete information. In this scenario, $p(x)>\frac{m(b,s)}{s}\eta x$ for all sellers-$(x,\delta=0)$ and thus they trade in centralized market. Moreover, sellers-$(x,\delta=1)$ would be indifferent between selling in centralized market at $t=1$ and waiting till $t=2$.

Secondly, in the presence of information asymmetry, $p(x)=p$ for all sellers pooling in centralized market. On one hand, for sellers with $\delta=0$, if decentralized market does not exist, their only choice is the centralized market. If the decentralized market exists, however, they would compare $\frac{m(b,s)}{s}\eta x$ with p. Furthermore, if $\frac{m(b,s)}{s}\eta x_1>p$, we would also have $\frac{m(b,s)}{s}\eta x_2>p$ provided $x_2>x_1$. Thus there may exist a cut-off point $\widetilde{x}$ on the choice of trading venues. If $\widetilde{x}\in(x_L,x_H)$, then $\frac{m(b,s)}{s}\eta\widetilde{x}=p$ holds by definition. On the other hand, for sellers with $\delta=1$, as argued above, they would never consider trading in decentralized market even though it would be available. Instead, they simply compare p and x. As a result, those with $x<p$ would sell their asset claims in the centralized market at $t=1$ while those with $x\geq p$ would enter either markets and wait till $t=2$.

Finally, based the above two pieces of observation, we have

$$\begin{aligned} U^s(x,\delta) &= \begin{cases} p & if\ \delta=0\ and\ x\leq\widetilde{x} \\ \frac{x}{\widetilde{x}}\cdot p & if\ \delta=0\ and\ x>\widetilde{x} \\ p & if\ \delta=1\ and\ x\leq p \\ x & if\ \delta=0\ and\ x\leq\widetilde{x} \end{cases} \\ &= \begin{cases} max\{\frac{x}{\widetilde{x}},\ 1\}\cdot p & if\ \delta=0 \\ max\{\frac{x}{p},\ 1\}\cdot p & if\ \delta=1 \end{cases} \\ &= max\{\frac{x}{p+(\widetilde{x}-p)\cdot\mathbf{1}_{\{\delta=0\}}},\ 1\}\cdot p \end{aligned}$$

Proof of Corollary 1: It is immediately obtained by using Proposition 1 and 2.

Proof of Lemma 1: *The results in the general case in proved as below.*
First of all, we show that $p \leq \widetilde{x}$. Eq. (4) suggests that

$$\begin{aligned} p &= \frac{\pi F(\widetilde{x})\mathbb{E}(x|x \leq \widetilde{x}) + (1-\pi)F(p)\mathbb{E}(x|x \leq p)}{\pi F(\widetilde{x}) + (1-\pi)F(p)} \\ &= \frac{\pi \int_{x_L}^{\widetilde{x}} xdF(x) + (1-\pi)\int_{x_L}^{p} xdF(x)}{\pi F(\widetilde{x}) + (1-\pi)F(p)} \\ &\leq \frac{\pi \int_{x_L}^{\widetilde{x}} \widetilde{x}dF(x) + (1-\pi)\int_{x_L}^{p} pdF(x)}{\pi F(\widetilde{x}) + (1-\pi)F(p)} \\ &= \frac{\pi\widetilde{x}F(\widetilde{x}) + (1-\pi)pF(p)}{\pi F(\widetilde{x}) + (1-\pi)F(p)}, \end{aligned}$$

where the inequality strictly holds iff $x > x_L$. Thus $p \leq \frac{\pi\widetilde{x}F(\widetilde{x})+(1-\pi)pF(p)}{\pi F(\widetilde{x})+(1-\pi)F(p)}$. Multiplying both side of this inequality with $\pi F(\widetilde{x}) + (1-\pi)F(p)$ and rearranging then yields $p \leq \widetilde{x}$, where the equality holds iff $\widetilde{x} = x_L (= p)$.

Secondly, Eq. (4) can be rewritten as

$$G(p, \widetilde{x}, \pi) \equiv \pi \int_{x_L}^{\widetilde{x}} xdF(x) + (1-\pi)\int_{x_L}^{p} xdF(x) - \pi pF(\widetilde{x}) - (1-\pi)pF(p) = 0.$$

Thus we have

$$\begin{aligned} G_p \equiv \frac{\partial G}{\partial p} &= -[\pi F(\widetilde{x}) + (1-\pi)F(p)] < 0 \\ G_{\widetilde{x}} \equiv \frac{\partial G}{\partial \widetilde{x}} &= \pi(\widetilde{x} - p)f(\widetilde{x}) > 0 \end{aligned}$$

According to Implicit Function Theorem, we have

$$\frac{dp}{d\widetilde{x}} = -\frac{G_{\widetilde{x}}}{G_p} => 0.$$

Thus we can denote the above result as $p = P_{AS}(\widetilde{x}, \pi)$, which is an increasing function of $\widetilde{x}$. Furthermore, since $\widetilde{x} \geq x_L$, we immediately have $p \geq x_L$. When $\widetilde{x} = x_L$, Eq. (2) is reduced as follows.

$$p = \frac{\int_{x_L}^{p} xdF(x)}{F(p)} = \mathbb{E}(x|x \leq p),$$

which is a classic problem of adverse selection by Akerlof (1970) and the unique solution is $p = x_L$. As a result, $\mathcal{P}_{AS}(\widetilde{x} = x_L) = x_L$ and thus $p \geq x_L$. So far we finish the proof that $x_L \leq p \leq \widetilde{x}$, where both inequality strictly holds if $\widetilde{x} > x_L$.

Moreover, we have

$$\frac{\partial G}{\partial \pi} = [\int_{x_L}^{\widetilde{x}} xdF(x) - pF(\widetilde{x})] - [\int_{x_L}^{p} xdF(x) - pF(p)]$$

Define $H(a;p) \equiv \int_{x_L}^{a} x dF(x) - pF(a)$. Then we have $\frac{\partial H}{\partial a} = (a-p)f(a)$ and thus $H(a;p)$ increases with a when $a > p$. Since $\widetilde{x} > p$, we have

$$G_\pi \equiv \frac{\partial G}{\partial \pi} = H(\widetilde{x};p) - H(p;p) > 0,$$

which in turn, by using Implicit Function Theorem again, implies that

$$\frac{dp}{d\pi} = -\frac{G_\pi}{G_p} > 0.$$

Denote $p = P_{AS}(\widetilde{x}, \pi)$. Thus $p = P_{AS}(\widetilde{x}, \pi) \leq P_{AS}(\widetilde{x}, \pi = 11) = \mathbb{E}(x|x \leq \widetilde{x}) \leq \mathbb{E}(x|x \leq x_H) = \mu(\theta)$. Finally, when $\pi = 0$, Eq. (4) is reduced to

$$p = \frac{\int_{x_L}^{p} x dF(x)}{F(p)} = \mathbb{E}(x|x \leq p),$$

which has been discussed above in the case when $\widetilde{x} = x_L$. The only solution is $p = P_{AS}(\widetilde{x}, \pi = 0) = x_L$ and CM totally collapses.

Now we prove the second part of Proposition 3.

When $x \overset{U}{\sim} X = [x_L, x_H]$, we have

$$F(x) = \begin{cases} 0 & if\ x < x_L \\ \frac{x - x_L}{x_H - x_L} & if\ x_L \leq x \leq x_H \\ 1 & if\ x > x_H \end{cases}.$$

Substituting $F(x)$ into Eq. (2) and making some algebraic manipulation yields Eq. (3).

Proof of Lemma 2: The results in the general case in proved as follows. For the ease of argument, we list Eq. (4) and Eq. (3) as below.

$$\begin{aligned} m(1,\alpha)r(\widetilde{x}) &= \frac{\kappa}{1-\eta} \\ m(\frac{1}{\alpha},1)\widetilde{x} &= \frac{p}{\eta} \end{aligned}$$

In the above simultaneous equations, $\widetilde{x}$ and α are endogenous variables while κ, p and η are exogenous. To prove $\frac{\partial \widetilde{x}}{\partial p} > 0$, we differentiate both sides of the above two equations. Then we have

$$\begin{pmatrix} m_2(1,\alpha)r(\widetilde{x}) & m(1,\alpha)r'(\widetilde{x}) \\ -\frac{m_1(\frac{1}{\alpha},1)}{\alpha^2}\widetilde{x} & m(\frac{1}{\alpha},1) \end{pmatrix} \begin{pmatrix} \frac{d\alpha}{dp} \\ \frac{d\widetilde{x}}{dp} \end{pmatrix} = \begin{pmatrix} 0 \\ \frac{1}{\eta} \end{pmatrix} \ (\#)$$

Since $m_1 > 0$, $m_2 > 0$ and $r'(\widetilde{x}) > 0$, Cramer rule immediately suggests that

$$\frac{d\widetilde{x}}{dp} = \frac{det\begin{pmatrix} m_2(1,\alpha)r(\widetilde{x}) & 0 \\ -\frac{m_1(\frac{1}{\alpha},1)}{\alpha^2}\widetilde{x} & \frac{1}{\eta} \end{pmatrix}}{det\begin{pmatrix} m_2(1,\alpha)r(\widetilde{x}) & m(1,\alpha)r'(\widetilde{x}) \\ -\frac{m_1(\frac{1}{\alpha},1)}{\alpha^2}\widetilde{x} & m(\frac{1}{\alpha},1) \end{pmatrix}} > 0\ and\ \frac{d\alpha}{dp} = \frac{det\begin{pmatrix} 0 & m(1,\alpha)r'(\widetilde{x}) \\ \frac{1}{\eta} & m(\frac{1}{\alpha},1) \end{pmatrix}}{det\begin{pmatrix} m_2(1,\alpha)r(\widetilde{x}) & m(1,\alpha)r'(\widetilde{x}) \\ -\frac{m_1(\frac{1}{\alpha},1)}{\alpha^2}\widetilde{x} & m(\frac{1}{\alpha},1) \end{pmatrix}} < 0\ (*)$$

Following the same strategy delivers that $\frac{d\widetilde{x}}{d\kappa} > 0$ and $\frac{d\widetilde{x}}{d\eta} < 0$.

Now we prove the second part of Lemma 2. It is immediately done with the assumption $x \overset{U}{\sim} X = [x_L, x_H]$ and $m(b,s) = \lambda \cdot min\{b,s\}$.

Proof of Proposition 3: We proceed with the strategy of guess-and-verify. Assume market coexistence can be supported, *i.e.*, $\widetilde{x} \in (x_L, x_H)$, then we have

$$\begin{aligned} \frac{\lambda \cdot min\{b,s\}}{b}(1-\eta)\mathbb{E}(x|x \geq \widetilde{x}) &= \kappa \\ \frac{\lambda \cdot min\{b,s\}}{s}\eta\widetilde{x} &= p \\ s &= \pi \cdot [1 - F(\widetilde{x})] \\ p &= \varphi\widetilde{x} + (1-\varphi)x_L \end{aligned}$$

Additionally, we assume that $s < b$, *i.e.*, $\alpha \equiv \frac{s}{b} < 1$, then the above equations suggest that $\widetilde{x} = (\frac{1-\varphi}{\lambda\eta-\varphi})x_L$. We now have to check whether the guess that $\alpha < 1$ is valid. It then is easy for us to check that $\alpha < 1$ when $\kappa \leq \underline{\kappa}$. Moreover, we have to guarantee that $\widetilde{x} = (\frac{1-\varphi}{\lambda\eta-\varphi})x_L \in (x_L, x_H)$, which is true if and only $\lambda\eta > \varphi$ and $\sigma > \widehat{\sigma}$.

Similarly, we assume $\widetilde{x} \in (x_L, x_H)$, but $\alpha \geq 1$. Then we can show this guess is true if and only $\lambda\eta > \varphi$ and $\sigma > \widehat{\sigma}$ and $\kappa \in (\underline{\kappa}, \overline{\kappa})$.

Finally, based on the above analysis, $\widetilde{x} = x_H$ would be the only equilibrium result if $\kappa \geq \overline{\kappa}$ when $\lambda\eta > \varphi$ and $\sigma > \widehat{\sigma}$, or, for all $\kappa \in \mathbb{R}_+$, we have $\lambda\eta \leq \varphi$ and $\sigma \leq \widehat{\sigma}$.

Proof of Proposition 4: First, based on Proposition 3, we have market coexistence both before and after government intervention. Moreover, since $\kappa_g = \kappa' < \kappa_b = \kappa$, we know from Proposition 3 that $\widetilde{x}(\kappa_g) \leq \widetilde{x}(\kappa_b)$ and thus $p(\kappa_g) \leq p(\kappa_b)$. Additionally, the decrease of information cost implies a more favorable extensive margin for sellers. Therefore, we know that

$$U(\widetilde{x}(\kappa_g)) = p(\kappa_g) \leq p(\kappa_b) = \frac{m(s(\kappa_b), b(\kappa_b))}{s(\kappa_b)}\eta\widetilde{x}(\kappa_b) \leq \frac{m(s(\kappa_g), b(\kappa_g))}{s(\kappa_g)}\eta\widetilde{x}(\kappa_b) = U(\widetilde{x}(\kappa_b)).$$

Since $U(x)$ increases with x, there exists a cut-off point $\widehat{x} \in (\widetilde{x}(\kappa_g), \widetilde{x}(\kappa_b))$ such that

$$U(x) \begin{cases} \leq p(\kappa_b) & if\ x \leq \widehat{x} \\ \geq p(\kappa_b) & if\ x \geq \widehat{x} \end{cases}.$$

B Robust Analysis

In our baseline model, we use random search to characterize search frictions in DM. Besides, we assume liquidity shock, δ, only adopts two mass points, zero and one. Thus buyers could *infer* only sellers with

$\delta = 0$ could show up in DM. We use this appendix to undertake two pieces of robust check. First, we revisit the model with directed search rather than random search in DM. That is, each buyer in DM only run submarket-x, where sellers-(x, δ) would meet buyers. Secondly, we treat the general case on liquidity shock, in which δ is continuously distributed over an interval, just like the asset payoff x. In the general case, buyers can only detect asset payoff x with costly information acquisition, but have idea on liquidity shock δ. Thus buyers in DM would instead launch optimal contract to extract true value of δ for those sellers self-selecting into DM. Finally, we have so far focused on an exchange economy, *i.e.*, all of potential sellers is exogenous endowed with one unit of asset at $t = 0$.

Directed Search

In direct search, each buyer with information investment only engage in certain submarket-x. Correspondingly, seller-(x, δ) directly trade with buyers there. Since in equilibrium buyers would be indifferent among different sub-markets in DM, the following equation holds for all seller-(x, δ) self-selecting into DM.

$$\frac{m(b(x), s(x))}{b(x)}(1 - \eta)x = \kappa, \tag{B.1}$$

which immediately implies that $\alpha(x) \equiv \frac{s(x)}{b(x)}$, the market tightness at submarket-x, increases with x. In turn, the expected revenue of entering DM by seller-(x, δ), $\frac{m(b(x),s(x))}{s(x)}\eta x$ increases with x, just as that in random search. As a result, the property of cut-off point on market participation is preserved in the case with directed search. Following the procedure in Section 2 and 3, we have the following results on equilibrium choice of trading venues with directed search.

Corollary Denote $\underline{\kappa}' \equiv \lambda(1 - \eta)(\frac{1-\varphi}{\lambda\eta-\varphi})x_L$, $\underline{\kappa} \equiv \frac{\lambda(1-\eta)}{2}[(\frac{1-\varphi}{\lambda\eta-\varphi})x_L + x_H]$ and $\overline{\kappa} \equiv \lambda(1 - \eta)x_H$.

1. **(Choice of Trading Venues)**

   (a) For sellers with $\delta = 0$, there exists a cut-off point $\widetilde{x}' \in [x_L, x_H]$ such that if $x \geq \widetilde{x}'$, they would self-select into DM, and enter CM otherwise at $t = 1$.

   (b) For sellers with $\delta = 1$, if $x < p$, they would choose CM, and if $x \geq p$, they would participate in neither DM nor CM at $t = 1$, but instead wait to consume at $t = 2$.

2. **(Characterization of Cut-off Value $\widetilde{x}'$)**

   (a) When $\lambda\eta > \varphi$ and $\sigma > \hat{\sigma} \equiv (\frac{1-\lambda\eta}{1+\lambda\eta-2\varphi})\mu$ (*i.e.*, $\lambda\eta > \varphi + (1-\varphi)\frac{x_L}{x_H}$), where $\varphi = \varphi(\pi) \equiv \frac{\sqrt{\pi}}{\sqrt{\pi}+1}$, we have $\underline{\kappa}' < \underline{\kappa} < \overline{\kappa}$ and

$$\begin{aligned}
\widetilde{x}' &= min\{x_H,\ max\{(\frac{1-\varphi}{\lambda\eta-\varphi})\cdot x_L,\ \frac{\kappa}{\lambda(1-\eta)}\}\} \\
&= \begin{cases} x_H & if\ \kappa > \overline{\kappa} \\ \frac{\kappa}{\lambda(1-\eta)} & if\ \underline{\kappa}' < \kappa \leq \overline{\kappa} \\ (\frac{1-\varphi}{\lambda\eta-\varphi})x_L & if\ \kappa \leq \underline{\kappa}' \end{cases}. \\
p &= \varphi(\pi)\cdot\widetilde{x}' + (1-\varphi(\pi))\cdot x_L.
\end{aligned}$$

(b) When $\lambda\eta \leq \varphi(\pi)$ or $\sigma \leq \hat{\sigma}$ ($i.e.$, $\lambda\eta \leq \varphi + (1-\varphi)\frac{x_L}{x_H}$), we have

$$\widetilde{x} = x_H\ for\ all\ \kappa \in \mathbb{R}_+.$$

Since $m(b(x), s(x)) = \lambda\cdot min\{b(x), s(x)\}$, Eq. (B.1) suggests that, to recover information investment, informed buyers in DM would only accept to trade with sellers with $x \geq \frac{\kappa}{\lambda(1-\eta)}$. Then following the proof strategy of Proposition 5 yields the desired results.

This corollary implies the main results in the benchmark with random search are still preserved in our robust check with directed search. We illustrate key results of this corollary in Figure 9. Two comments are made here. First of all, the pattern of venue choice is qualitatively the same between random and directed search. Secondly, $\widetilde{x}' \leq \widetilde{x}$, $i.e.$, the size of DM tends to smaller under directed search than that under random search. As suggested in Section 4, a smaller DM would be save more social resources. Therefore our result is complementary to the findings by Moen (1997) on efficiency obtained by directed search.

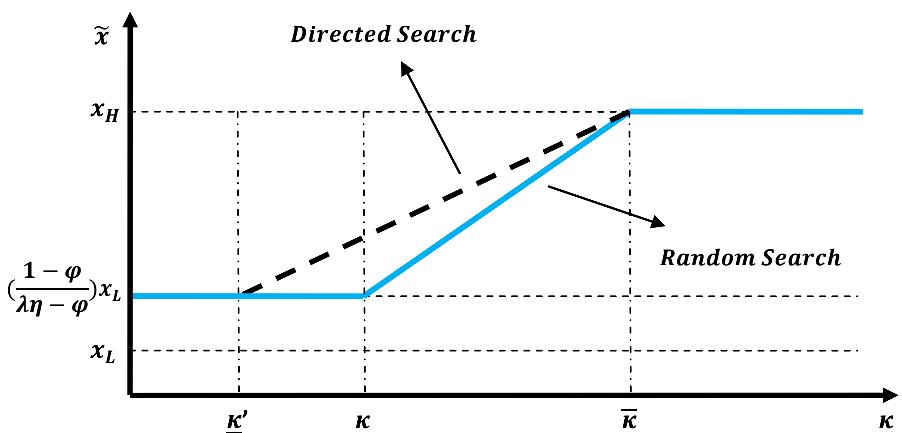

Figure 9: **Choice of Trading Venues: Direct vs Random Search**

Optimal Contract with a Continuum of δ

Now we return to random search with matching function $m(b,s) = \lambda \cdot min\{b,s\}$. The second line of model extension on switching from $\delta \in \{0,1\}$ to $\delta \in [0,1]$. Notice that buyers can no longer infer the true value of δ. Now we assign all bargaining power to buyers in DM and they could initiate optimal contract $\{q(x,\delta), \tau(x,\delta)\}$ after paying information cost κ in DM. Given any x, $q(x,\delta)$ and $\tau(x,\delta)$ denote the quantity of asset transferred to buyers and the consumption paid to sellers respectively if sellers report his type of private value as δ. Note that x is verifiable after buyers incurring information cost κ.

In the same spirit in the benchmark, there exists a cut-off value of κ, say $\overline{\kappa}'$, above which DM cannot not be supported whatever the contract buyers propose in DM. In contrast, the equilibrium with market coexistence is not only sustainable, but also stable if $\kappa < \overline{\kappa}^*$. Moreover, there exists another cut-off point $\underline{\kappa}^* < \overline{\kappa}^*$ such that $b < s$ in equilibrium if $\kappa < \underline{\kappa}^*$. Since the coexistence of CM and DM is the most intriguing part, we assume $\kappa < \overline{\kappa}^*$ holds. Moreover, to focus on the characterization of optimal contract by buyers in DM, we assume $\kappa < \underline{\kappa}^* < \overline{\kappa}^*$ throughout this subsection such that DM can not only be supported, but also there are more buyers than sellers flowing into DM. We can prove that the qualitative results shown below are still held if $\kappa \in \underline{\kappa}^* < \overline{\kappa}^*$ (and $0 < b < s$ correspondingly).

Denote $U(x,\delta)$ as the gain of seller-(x,δ) by enrolling in the contract by buyers in DM. Now seller-(x,δ) makes her discrete choice among three alternatives.

$$max\{\underbrace{p}_{CM}, \underbrace{\frac{m(b,s)}{s} \cdot U(x,\delta) + [1 - \frac{m(b,s)}{s}] \cdot \delta x}_{DM}, \underbrace{\delta x}_{No-trade}\}.$$

In this part, we focus on the most intriguing case, *i.e.*, the coexistence between CM and DM. Thanks to Revelation Principle, given any x, we could simply focus on buyer's direct mechanism in DM, which is formulated as below.

$$\Pi_B(x) \equiv \max_{\{q(x,\delta)\in[0,1],\tau(x,\delta)\in[0,\infty)\}_{\mathbf{Z}_{DM}|x}} \{\int_{\delta\in\mathbf{Z}_{DM}|x} [-\tau(x,\delta) + q(x,\delta)\cdot x]\}$$

subject to

$$\begin{aligned} U(x,\delta) &\equiv U_x(\delta;\delta) = \max_{\delta'\in\mathbf{Z}_{DM}|x}\{U_x(\delta;\delta')\} \\ U_x(\delta;\delta') &\equiv [1 - q(x,\delta')]\cdot\delta x + \tau(x,\delta')\ (IC) \\ \frac{m(b,s)}{s}\cdot U(x,\delta) + [1 - \frac{m(b,s)}{s}]\cdot\delta x &\geq \underline{\mathbf{U}}(\mathbf{p},\boldsymbol{\delta}\mathbf{x}) \equiv \mathbf{max}\{\mathbf{p},\ \boldsymbol{\delta}\mathbf{x}\}\ (IR). \end{aligned}$$

Similar to standard mechanism design, both Incentive Compatibility (IC) and Individual Rationality (IR) should be satisfied. What makes our setup challenging is that, buyer's mechanism design would affect $\mathbf{Z}_{DM}$, the content of seller-(x,δ) self-selecting into the contracts in DM. Moreover, it is worth noting the outside option is type-dependent and thus may involve in the so-called countervailing incentive *a la* Lewis and Sappington (1989), Maggi and Rodriguez-Clare (1995) and Jullien (2000).

The following lemma on seller's choice of trading venues generalizes the results of Proposition 2.

Lemma (Seller's Choice of Trading Venues) For any seller-(x, δ), given p in CM and contract $\{q(x, \delta), \tau(x, \delta)\}$ in DM, there exists cut-off values $\underline{\delta}(x)$ and $\overline{\delta}(x)$ such that,

1. if $\delta \in [0, \underline{\delta}(x)]$, she sells her asset at CM;
2. if $\delta \in (\underline{\delta}(x), \overline{\delta}(x)]$, she sells her asset at DM;
3. if $\delta \in (\overline{\delta}(x), 1]$, she chooses not to trade.

We rewrite the IR condition of this mechanism design as below.

$$U(x, \delta) \geq V(x, \delta, p) \equiv \delta x + \frac{1}{\widehat{\lambda}} max\{p - \delta x, 0\},$$

where $\widehat{\lambda} \equiv \frac{m(b,s)}{s}$ and thus $V(x, \delta, p)$ decreases with δ when $\delta x < p$ while increases when $\delta x > p$. The non-monotone property of V stems from the fact that, relative to DM, seller-(δ, x) have two outside options. One is sell at CM at price p while the other one is pure autarky, *i.e.*, participating in neither CM nor DM. When δx is low, the outside option with CM is larger than that in with autarky. It is just opposite when δx is high enough.

Secondly, given any x, Envelope Theorem suggests that

$$\frac{\partial U(x, \delta)}{\partial \delta} = [1 - q(x, \delta)]x \geq 0.$$

Combining these two observations yields the results in the lemma with

$$\begin{aligned} \underline{\delta}(x) &= max\{0, \frac{p - \widehat{\lambda} \cdot U(x, \delta)}{(1 - \widehat{\lambda}) \cdot x}\} \\ \overline{\delta}(x) &= min\{1, \frac{U(x, \delta)}{x}\} \end{aligned}$$

Finally,a figure with δ in horizontal axis and U, V in vertical axis would help illustrate our findings. Due to space concern, we omit it here.

Based on the above lemma, we reach the optimal contract in DM and in turn obtain the explicit solution on $(\underline{\delta}(x), \overline{\delta}(x))$.

Proposition ***(Optimal Contract Offered in DM)*** *When* $\delta \overset{U}{\sim} \Delta = [0, 1]$ *and* $m(b, s) = \lambda \cdot min\{b, s\}$ *and* κ *is small enough, given any* x *in DM, buyer's optimal contract is in the form as take-it-or-leave-it in the following form.*

$$\{q^*(x, \delta)) = 1, \tau^*(x, \delta) = \tau(x)\},$$

where

$$\tau(x) = \begin{cases} \frac{p+x}{2} & if\ x \in [p, \frac{2-\lambda}{\lambda} \cdot p] \\ \frac{p}{\lambda} & if\ x \in (\frac{2-\lambda}{\lambda} \cdot p, \frac{2\cdot}{\lambda} p] \\ \frac{x}{2} & if\ x \in (\frac{2}{\lambda} \cdot p, x_H] \end{cases}.$$

In turn, we have

$$\underline{\delta}(x) = max\{0, \frac{p - \lambda \cdot \tau(x)}{(1-\lambda) \cdot x}\} = \begin{cases} 1 & if\ x \in [x_L, p] \\ \frac{(1-\frac{\lambda}{2})p - \frac{\lambda}{2} x}{(1-\lambda)x} & if\ x \in (p, \frac{2-\lambda}{\lambda} \cdot p] \\ 0 & if\ x \in (\frac{2-\lambda}{\lambda} \cdot p, \frac{2}{\lambda} \cdot p] \\ 0 & if\ x \in (\frac{2}{\lambda} \cdot p, x_H] \end{cases}.$$

$$\overline{\delta}(x) = min\{1, \frac{\tau(x)}{x}\} = \begin{cases} 1 & if\ x \in [x_L, p] \\ \frac{p+x}{2x} & if\ x \in (p, \frac{2-\lambda}{\lambda} \cdot p] \\ \frac{p}{\lambda x} & if\ x \in (\frac{2-\lambda}{\lambda} \cdot p, \frac{2}{\lambda} \cdot p] \\ \frac{1}{2} & if\ x \in (\frac{2}{\lambda} \cdot p, x_H] \end{cases}.$$

The first part is proved as below.

To ease illustration while preserving the key insights, we have assumed that information cost κ is low enough such that $b > s$ is always true in equilibrium. In turn, we have $\frac{m(b,s)}{s} = \lambda \in (0,1)$, a constant. This would help us focus on characterizing optimal contract by buyers. We break down the proof into the following steps.

First of all, since sellers could always seller their assets at price p in CM and $\delta \in [0,1]$, buyers in DM would have no customers if $U(x,\delta) < p$. Meanwhile, to recover information cost, buyers in DM ex ante would never accept sellers with $x < p$.

Secondly, for seller-(x,δ) self-selecting into DM and is allowed to trade with buyers there, denote $\widetilde{\delta}(x) = min\{\frac{p}{x}, 1\} = \frac{p}{x}$. Since Then we can check that $\underline{\delta}(x) \leq \widetilde{\delta}(x) \leq \overline{\delta}(x)$, where $\underline{\delta}(x)$ and $\overline{\delta}(x)$ are characterized in the proof of Lemma 1. For each x, buyers launch direct mechanism for two groups respectively. One is $\delta \in \Delta_1 = [\underline{\delta}(x), \widetilde{\delta}(x)]$ while the other group is $\delta \in \Delta_2 = [\widetilde{\delta}(x), \overline{\delta}(x)]$. On one hand, for each group, buyers make sure IR and IC conditions are satisfied. On the other hand, buyers have to make sure sellers in group $\Delta_1 \cup \Delta_2$ would have no incentive to deviate the other group. After all, even though x is verifiable after buyers incur information cost, δ is still unobservable. As a result, incentive compatibility of not deviating to another group has to be additionally taken into account. In the next two pieces of analysis, we first solve the within-group contract and then go to discussion of IC on across-group.

Buyer's objective function for group Δ_1 is

$$\Pi_B(x)|_{\Delta_1} \equiv \max_{\{q(x,\delta)\in[0,1], \tau(x,\delta)\in[0,\infty)\}} \{\int_{\underline{\delta}(x)}^{\widetilde{\delta}(x)} [-\tau(x,\delta) + q(x,\delta) \cdot x]\}$$

Meanwhile, for group with $\delta \in \Delta_1$, the outside option is simplified as $V(x,\delta,p) \equiv \delta x + \frac{1}{\lambda} max\{p - \delta x, 0\} = \frac{p}{\lambda} - (\frac{1}{\lambda} - 1)\delta x$. That is, for sellers in this group, the outside option decreases with δ. Following Maggi and Rodriguez-Clare (1995), among others, we define $\Upsilon(x,\delta) = U(x,\delta) - V(x,\delta,p)$. Envelope Theorem suggests

$$\frac{\partial \Upsilon}{\partial \delta} = [\frac{1}{\lambda} - q(x,\delta)] \cdot x.$$

Thus

$$[1 - q(x,\delta)]\delta x + \tau(x,\delta) - [\frac{p}{\lambda} - (\frac{1}{\lambda} - 1)\delta x] = \int_{\underline{\delta}(x)}^{\delta} [\frac{1}{\lambda} - q(x,\delta')] x d\delta'.$$

Expressing the above equation for $\tau(x,\delta)$ and substituting it into the buyer's objective function for group Δ_1 mentioned above, we can easily prove that, for group Δ_1, $q^*(x,\delta)|_{\Delta_1} = 1$. Substituting it into the above equation suggests that $\tau^*(x,\delta)|_{\Delta_1}$ has nothing to with δ and is thus denoted as $\tau^*(x)|_{\Delta_1}$.

Similarly, we can show that $q^*(x,\delta)|_{\Delta_2} = 1$ and $\tau^*(x,\delta)|_{\Delta_2}$ also has nothing to do with δ and is thus denoted as $\tau^*(x)|_{\Delta_2}$. Finally, to make sure the IC condition of across-group is satisfied, we have to make sure $\tau^*(x,\delta)|_{\Delta_1} = \tau^*(x,\delta)|_{\Delta_1} \equiv \tau(x)|_{\Delta_1 \cup \Delta_2} = \tau(x)$. In sum, given $x > p$ and buyers and sellers are matched in DM, the optimal contract would take the form as $\{q^*(x,\delta) = 1, \tau^*(x,\delta) = \tau(x)\}$. It is obvious that $\tau(x) \leq x$ is always held.

In turn, we have $U(x,\delta) = \tau(x)$ and thus

$$\begin{aligned} \underline{\delta}(x) &= max\{0, \frac{p - \lambda \cdot \tau(x)}{(1-\lambda) \cdot x}\} \\ \overline{\delta}(x) &= min\{1, \frac{\tau(x)}{x}\} = \frac{\tau(x)}{x}. \end{aligned}$$

As a recap, buyer's profit function focusing on sellers with x is

$$\Pi_B(x) \equiv \max_{\{q(x,\delta)\in[0,1],\tau(x,\delta)\in[0,\infty)\}_{\mathbf{Z}_{DM}|x}} \{\int_{\delta \in \mathbf{Z}_{DM}|x} [-\tau(x,\delta) + q(x,\delta) \cdot x]\}$$

Using the optimal contract and cut-off values just obtained above, $\Pi_B(x)$ is refined as below.

$$\Pi_B(x) = \max_{\tau(x,\delta)\in[0,x]} [x - \tau(x)][G(\overline{\delta}(x)) - G(\underline{\delta}(x))]$$

subject to

$$\begin{aligned} \underline{\delta}(x) &= max\{0, \frac{p - \lambda \cdot \tau(x)}{(1-\lambda) \cdot x}\} \\ \overline{\delta}(x) &= min\{1, \frac{\tau(x)}{x}\} = \frac{\tau(x)}{x}, \end{aligned}$$

where G denotes the CDF of δ with support $[0,1]$. If we further assume $\delta \overset{U}{\sim} \Delta = [0,1]$, then we obtain $\tau(x)$ as that in Proposition 5. In turn, we obtain $\underline{\delta}(x)$ and $\overline{\delta}(x)$ as in the second part of this proposition. We are done.

We illustrate Proposition 5 in Figure 10 and 11 respectively the terms of trade by buyers and choice of trading venues by sellers.

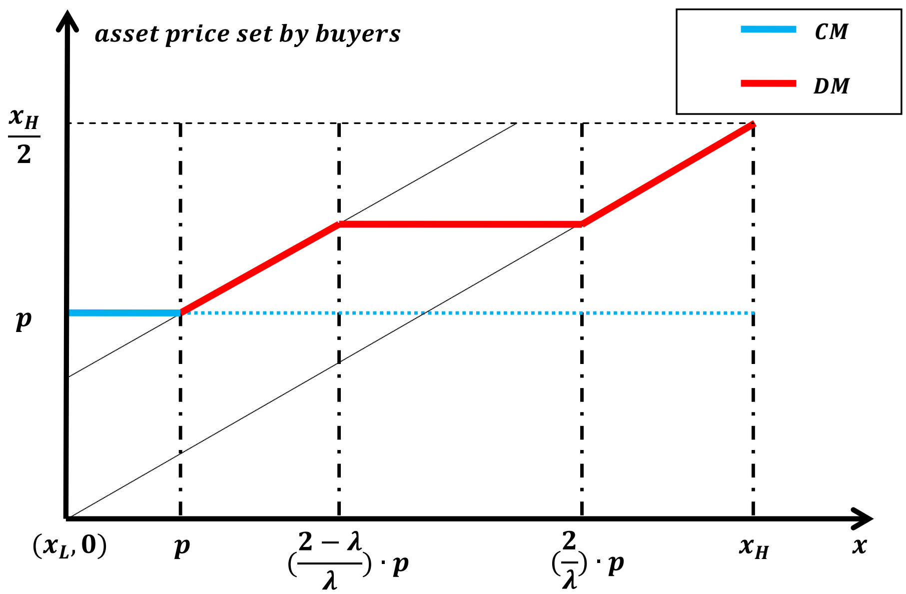

Figure 10: $(p, \tau(x))$**: Assets Prices in CM and DM**

Several remarks are made here. First of all, the optimal contract by buyers in DM only focus on sellers with $x > p$. On one hand, p is always an outside option of any seller-(x, δ) and thus buyers in DM would attract no sellers if $\tau(x, \delta) < p$. On the other hand, buyer's profit is $x - \tau(x, \delta)$. To make the profit non-negative, it must that they would trade with $x > p$ and x can always be verifiable. Secondly, given price in CM p, seller's choice over CM, DM and autarky not depends on common value x, but also on private value δ. Thirdly, we are still in the position of partial equilibrium since price in CM is taken as given. Based on Proposition 7, p is solved in equilibrium as below.

$$p = \frac{\int_{x_L}^{p} x dF(x) + \int_{p}^{min\{\frac{2-\lambda}{\lambda} \cdot p, x_H\}} x G\left(\frac{(1-\frac{\lambda}{2})p - \frac{\lambda}{2}x}{(1-\lambda)x}\right) dF(x)}{F(p) + \int_{p}^{min\{\frac{2-\lambda}{\lambda} \cdot p, x_H\}} G\left(\frac{(1-\frac{\lambda}{2})p - \frac{\lambda}{2}x}{(1-\lambda)x}\right) dF(x)}, \tag{B.2}$$

where F and G denotes the CDF of x and δ respectively. Moreover, we have implicitly assumed $G(\delta)$ is a uniform distribution. However, even though $F(x)$ is a uniform distribution, the above equation has no explicit solution on p.

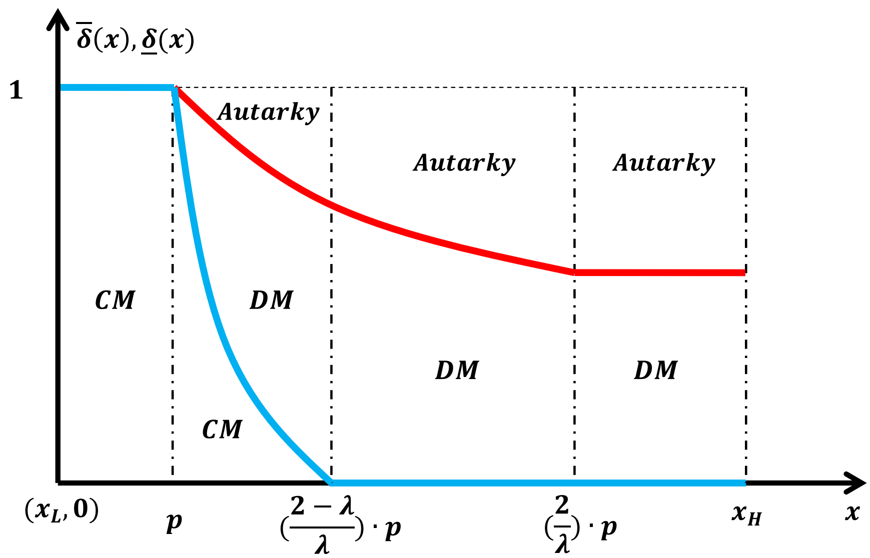

Figure 11: **Seller's Choice of Trading Venues**

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